Semilattices

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- Each semilattice $(A, \leq)$ has a binary *operation* $\circ$ with $a \circ b = \inf\{a, b\}$ $(\sup\{a, b\})$ for $a, b \in A$.
- if $\circ$ is a binary operation on a set A which satisfies

$$a \circ (b \circ c) = (a \circ b) \circ c, a \circ b = b \circ a, a \circ a = a,$$

then on the set A we may define a partial ordering $\leq$ by

$$a \leq b \Leftrightarrow a \circ b = a \; and \; \inf\{a, b\} = a \circ b$$

where $(A, \leq)$ is a semilattice.

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- $co(\leq^A)$-collection of convex orderings with respect to $\leq^A$.

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Bounded branching

Let $m \geq 1$ be a natural number. We consider a sequence $(R_{m,i})_{i=1}^{m}$ of binary relational symbols and the class $\mathcal{DT}_m$ which contains structures of the form $(A, \circ^A, \leq^A, (R_{m,i}^A)_{i=1}^m)$ where:

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- $(A, \circ^A, \leq^A) \in \mathcal{T}_m$,
- $R_{m,i}^A(a,b) \Rightarrow a <^A b$,
- $R_{m,i}^A(a,b), b \leq^A c \Rightarrow R_{m,i}^A(a,c)$,
- $R_{m,i}^A(a,b), b \circ^A c = a \Rightarrow \rceil R_{m,i}^A(a,c)$,
- $a <^A b \Rightarrow (\exists i)[R_{m,i}^A(a,b)]$.

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where $a_1, ..., a_n$ are immediate successors of the root, structures $\mathbb{A}(a_1), ..., \mathbb{A}(a_k)$ are mutually non isomorphic,for every $j > k$ there is $i \leq k$ such that $\mathbb{A}(a_i) \cong \mathbb{A}(a_j)$ and for $i \leq k$ we have

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Hrushovski class$\longrightarrow$unique ergodicity

Theorem

$Aut(\mathbb{T})$ and $Aut(\mathbb{T}_m)$ are uniquely ergodic groups which are not extremely amenable and $\mathcal{T}$ and $\mathcal{T}_m$ are not Hrushovski classes.

Thank you