Smart Power Systems, Renewable Energies and Markets: the Optimization Challenge

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In 2000, the Optimization and Systems team was created at École des Ponts ParisTech and, since then, we have trained PhD students in stochastic optimization, mostly with Électricité de France Research and Development.

Since 2011, we witness a growing demand from energy firms for stochastic optimization, fueled by a deep and fast transformation of power systems.

We discuss to what extent optimization is challenged by the transformation of power systems driven by renewable energies penetration, telecommunication technologies and markets.

More renewable energies → more unpredictability + more variability →
- more storage → more dynamic optimization, optimal control
- more stochastic optimization

hence, stochastic optimal control

We shed light on the two main new issues in stochastic control in comparison with deterministic control: risk attitudes and online information.

We cast a glow on two snapshots highlighting ongoing research in the field of stochastic control applied to energy.
Outline of the presentation

1. Long term industry-academy cooperation
2. The transformation of power systems seen from an optimizer perspective
3. Moving from deterministic to stochastic dynamic optimization
4. Two snapshots on ongoing research
5. A need for training and research
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   - École des Ponts ParisTech–Cermics–Optimization and Systems
   - Industry partners

2. The transformation of power systems seen from an optimizer perspective
   - The transformation of power systems
   - Optimization is challenged

3. Moving from deterministic to stochastic dynamic optimization
   - Dam models
   - The deterministic optimization problem is well posed
   - In the uncertain framework, the optimization problem is not well posed
   - Ingredients for stochastic dynamic optimization problems

4. Two snapshots on ongoing research
   - Decomposition-coordination optimization methods under uncertainty
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5. A need for training and research
École des Ponts ParisTech

- The École nationale des ponts et chaussées was founded in 1747 and is one of the world’s oldest engineering institutes
- École des Ponts ParisTech is traditionally considered as belonging to the 5 leading engineering schools in France
- Young graduates find positions in professional sectors like transport and urban planning, banking, finance, consulting, civil works, industry, environnement, energy, etc.
- Faculty and staff
  - 217 employees (including 50 subsidaries).
  - 165 module leaders, including 68 professors.
  - 1509 students.
- École des Ponts ParisTech is part of University Paris-Est
Research at École des Ponts ParisTech

- Figures on research
  - Research personnel: 220
  - About 40 École des Ponts PhDs students graduate each year

- 10 research centers
  * CEREA (atmospheric environment), joint École des Ponts-EDF R&D
  * CEREVE (water, urban and environment studies)
  * CERMICS (mathematics and scientific computing)
  * CERTIS (information technologies and systems)
  * CIRED (international environment and development)
  * LATTS (techniques, regional planning and society)
  * LVMT (city, mobility, transport)
  * UR Navier (mechanics, materials and structures of civil engineering, geotechnic)
  * Saint-Venant laboratory (fluid mechanics), joint École des Ponts-EDF R&D
  * Paris School of Economic PSE
CERMICS, Centre d’enseignement et de recherche en mathématiques et calcul scientifique

- The scientific activity of CERMICS covers several domains in
  - scientific computing
  - modelling
  - optimization
- 14 senior researchers
  - 14 PhD
  - 11 habilitation à diriger des recherches
- Three missions
  - Teaching and PhD training
  - Scientific publications
  - Contracts
- 550,000 euros of contracts per year with
  - research and development centers of large industrial firms: CEA, CNES, EADS, EDF, Rio Tinto, etc.
  - public research contracts
Optimization and Systems Group

- Three senior researchers
  - J.-P. CHANCELIER
  - M. DE LARA
  - F. MEUNIER

- Five PhD students
  - J.-C. ALAIS (CIFRE EDF)
  - S. SEPULVEDA (foreign co-supervision)
  - V. LECLÈRE (IPEF)
  - T. PRADEAU
  - P. SARRABEZOLLES

- Four associated researchers
  - P. CARPENTIER (ENSTA ParisTech)
  - L. ANDRIEU (EDF R&D)
  - K. BARTY (EDF R&D)
  - A. DALLAGI (EDF R&D)
Optimization and Systems Group research specialities

**Methods**
- Stochastic optimal control (discrete-time)
  - Large-scale systems
  - Discretization and numerical methods
  - Probability constraints
- Discrete mathematics; combinatorial optimization
- System control theory, viability and stochastic viability
- Numerical methods for fixed points computation
- Uncertainty and learning in economics

**Applications**
- Optimized management of power systems under uncertainty
  (production scheduling, power grid operations, risk management)
- Transport modelling and management
- Natural resources management (fisheries, mining, epidemiology)

**Softwares**
- Scicoslab, NSP
- Oadlibsim
Publications since 2000

- 24 publications in peer-reviewed international journals
- 3 publications in collective works
- 3 books
  - Modeling and Simulation in Scilab/Scicos with ScicosLab 4.4 (2e édition, Springer-Verlag)
  - Introduction à SCILAB (2e édition, Springer-Verlag)
  - Sustainable Management of Natural Resources. Mathematical Models and Methods (Springer-Verlag)
- 2 books to be finished soon:
  - *Stochastic Optimization. At the Crossroads between Stochastic Control and Stochastic Programming*
  - *Control Theory for Engineers*
Teaching

- **Masters**
  - *Mathématiques, Informatique et Applications*
  - *Économie du Développement Durable, de l'Environnement et de l'Énergie*
  - *Renewable Energy Science and Technology Master ParisTech*

- **École des Ponts ParisTech**
  - Optimisation et contrôle (J.-P. CHANCELIER)
  - Modéliser l'aléa (J.-P. CHANCELIER)
  - Introduction à Scilab (J.-P. CHANCELIER, M. DE LARA)
  - Modélisation pour la gestion durable des ressources naturelles (M. DE LARA)
  - Économie du risque, effet de serre et biodiversité (M. DE LARA)
Industrial and public contracts since 2000

- **Industrial contracts**
  - Conseil français de l’énergie (CFE)
  - SETEC Energy Solutions
  - Électricité de France (EDF R&D)
  - Thales
  - Institut français de l’énergie (IFE)
  - Gaz de France (GDF)
  - PSA

- **Public contracts**
  - STIC-AmSud (CNRS-INRIA-Affaires étrangères)
  - Centre d’étude des tunnels
  - CNRS ACI Écologie quantitative
  - RTP CNRS
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We cooperate with industry partners

- As academics, we cooperate with industry partners, looking for long-lasting close relations.
- We are not consultants working for clients.
- Our job consists mainly in:
  - Training Master and PhD students, working in the company and interacting with us, on subjects designed jointly.
  - Developing methods, algorithms.
  - Contributing to computer codes developed within the company.
  - Training professional engineers in the company.
Électricité de France R & D / Département OSIRIS

- Électricité de France is the French electricity main producer
  - 159 000 collaborateurs dans le monde
  - 37 millions de clients dans le monde
  - 65,2 milliards d’euros de chiffre d’affaire
  - 630,4 TWh produits dans le monde

- Électricité de France Research & Development
  - 486 millions d’euros de budget
  - 2 000 personnes

- Département OSIRIS
  Optimization, simulation, risks and statistics for the energy markets
  - 145 salariés (dont 10 doctorants)
  - 25 millions d’euros de budget
The Optimization and Systems Group has trained 8 PhD from 2004 to 2011 + 2 PhD students, the majority related with EDF and energy management.

* Laetitia ANDRIEU, former PhD student at EDF, now researcher EDF
* Kengy BARTY, former PhD student at EDF, now researcher EDF
* Daniel CHEMLA, former PhD student
* Anes DALLAGI, former PhD student at EDF, now researcher EDF
* Laurent GILOTTE, former PhD student with IFE, researcher EDF
* Pierre GIRARDEAU, former PhD student at EDF, now with ARTELYS
* Eugénie LIORIS, former PhD student
* Babacar SECK, former PhD student at EDF
* Cyrille STRUGAREK, former PhD student at EDF, now with Munich-Ré
* Jean-Christophe ALAIS, PhD student at EDF
* Vincent LECLERE, PhD student (partly at EDF)
PhD subjects

- Contributions to the Discretization of Measurability Constraints for Stochastic Optimization Problems,
- Optimization under Probability Constraint,
- Uncertainty, Inertia and Optimal Decision. Optimal Control Models Applied to Greenhouse Gas Abatement Policies Selection,
- Variational Approaches and other Contributions in Stochastic Optimization,
- Particular Methods in Stochastic Optimal Control,
- From Risk Constraints in Stochastic Optimization Problems to Utility Functions,
- Resolution of Large Size Problems in Dynamic Stochastic Optimization and Synthesis of Control Laws,
- Evaluation and Optimization of Collective Taxis Systems,
- Risk and Optimization for Energies Management,
- Risk, Optimization, Large Systems,
Other contacts with French small consulting companies

- **ARTELYS** is a company specializing in optimization, decision-making and modeling. Relying on their high level of expertise in quantitative methods, the consultants deliver efficient solutions to complex business problems. They provide services to diversified industries: Energy & Environment, Logistics & Transportation, Telecommunications, Finance and Defense.

- Créée en 2011, **SETEC Energy Solutions** est la filiale du groupe SETEC spécialisée dans les domaines de la production et de la maîtrise de l’énergie en France et à l’étranger. SETEC Energy Solutions apporte à ses clients la maîtrise des principaux process énergétiques pour la mise en œuvre de solutions innovantes depuis les phases initiales de définition d’un projet jusqu’à son exploitation.
Summary

The following slides on the transformation of power systems express a viewpoint from an optimizer perspective working in an optimization research group in an applied mathematics research center in a French engineering institute having contributed to train students now working in energy having contacts and contracts with energy/environment firms
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Three key drivers for the transformation of power systems

- Environment
- Markets
- Technology
Key driver: environmental concern

- Strong impulse towards a decarbonized energy system
- Expansion of renewable energy sources

⇒ Successfully integrating renewable energy sources has become critical, and made especially difficult due to their unpredictable and highly variable nature
Key driver:

economic deregulation

- A power system (generation/transmission/distribution)
  - less and less vertical (deregulation of energy markets)
  - hence with many players with their own goals
- with some new players
  - industry (electric vehicle)
  - regional public authorities (autonomy, efficiency)
- with a network in horizontal expansion
  (the Pan European system counts 10,000 buses, 15,000 power lines, 2,500 transformers, 3,000 generators, 5,000 loads)
- with more and more exchanges (trade of commodities)

⇒ A change of paradigm for management
from centralized to more and more decentralized
Key driver:
telecommunication, metering and computing technology

A power system with more and more technology
due to evolutions in the fields of computing and telecoms

- smart meters
- sensors
- controllers
- grid communication devices, etc.

⇒ A huge amount of data which, one day, will be
a new potential for optimized management
The “smart grid”? An infrastructure project with promises to be fulfilled by a “smart power system”

- **Hardware / infrastructures / smart technologies**
  - Renewable energies technologies
  - Smart metering
  - Storage

- **Promises**
  - Quality, tariffs
  - More safety
  - More renewables (environmentally friendly)

- **Software / smart management**
  (energy supply being less flexible, make the demand more flexible)
  
  smart management, smart operation, smart meter management, smart distributed generation, load management, advanced distribution management systems, active demand management, diffuse effacement, distribution management systems, storage management, smart home, demand side management, etc.
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A call from the industry to optimizers

- Every three years, Électricité de France (EDF) organizes an *international Conference on Optimization and Practices in Industry* (COPI)
- At the last COPI’11, Jean-François Faugeras from EDF R&D opened the conference with a plenary talk entitled “Smart grids: a wind of change in power systems and new opportunities for optimization”
- He claimed that “power system players are facing high level problems to solve requiring new optimization methods and tools”, with “not only a 'smart(er)' grid but a 'smart(er)' power system” and called on the optimizers to develop new methods
- In 2012, EDF R&D has sponsored a new program *Gaspard Monge pour l’Optimisation et la recherche opérationnelle* (PGMO) to support academic research in the field of optimization
What is “optimization”?

Optimizing is obtaining the best compromise between needs and resources

Marcel Boiteux (président d’honneur d’Électricité de France)

- **Resources**: portfolio of assets
  - production units
    - costly/not costly: thermal/hydropower
    - stock/flow, predictable/unpredictable: thermal/wind
  - tariffs options, contracts

- **Needs**: energy, safety, environment
  - energy uses
  - safety, quality, resilience (breakdowns, blackout)
  - environment protection (pollution) and alternative uses (dam water)

- **Best compromise**: minimize socio-economic costs (including externalities)
Electrical engineers métiers and skills are evolving

- **Unit commitment**, optimal dispatch of generating units: finding the least-cost dispatch of available generation resources to meet the electrical load
  - which unit? 0/1 variables
  - which power level? continuous variables

subject to more unpredictable energy flows (solar, wind) and demand (electrical devices, cars)

- **Markets**: day-ahead, intra-day (balancing market): dispatcher takes bids from the generators, demand forecasts from the distribution companies and clears the market

subject to more unpredictability, more players

- **Long term planning**

subject to more unpredictability (technologies, climate), more players

... Without even speaking of voltage, frequency and phase control
The transformation of power systems seen from an optimizer perspective

Optimization is challenged

Economic dispatch (static) as a cost-minimization problem under supply-demand balance

Consider energy production units \( i = 1, \ldots, N \), like coal, gas, nuclear, etc.

\[
\min_{(u_1, \ldots, u_N)} \sum_{i=1}^{N} J_i(u_i) \quad \text{under} \quad \sum_{i=1}^{N} \Theta_i(u_i) = D
\]

where

- \( u_i \) is the decision (production level) made for each unit \( i \)
- \( J_i(u_i) \) is the cost of making decision \( u_i \) for unit \( i \)
- \( \Theta_i(u_i) \) is the production induced by making decision \( u_i \) for unit \( i \)
- \( D \) is the demand
Economic dispatch (static) is made more delicate under uncertainty

\[
\min_{(u_1, \ldots, u_N)} \mathbb{E}\left[ \sum_{i=1}^{N} J_i(u_i, p_i) \right] \quad \text{under} \quad \sum_{i=1}^{N} \Theta_i(u_i, w_i) = D
\]

Mathematical description of sources of uncertainties (prices, failures, weather, demand, etc.):
- statistics? bounds?

Mathematical formulation of the criterion under uncertainty:
- in expectation? worst case?

Mathematical formulation of the constraints under uncertainty:
Optimization skills will follow the power system evolution

We focus on generation and trading, not on transmission and distribution

- Less base production and more wind and photovoltaic fatal generation makes supply more unpredictable
  → stochastic optimization
- Hence more storage (batteries, pumping stations)
  → dynamical optimization, reserves dimensioning
- The shape of the load is changing due to electric vehicle penetration
  → demand-side management, “peak shaving”, adaptive tariffs
- New subsystems emerge with local information and means of action: smart meters, new producers, micro-grid, virtual power plant
  → agregation, coordination, decentralized optimization
- Markets (day-ahead, intra-day)
  → optimization under uncertainty
- Environmental constraints on production (CO2) and resources usages (water)
  → risk constraints
The transformation of power systems seen from an optimizer perspective

Optimization is challenged

Summary

- Three major key factors — environmental concern, deregulation, telecommunication, metering and computing technology — drive the power systems mutation
- This induces a change of paradigm for management: from vertical centralized predictable “stock” energies to more horizontal decentralized unpredictable variable “flow” energies
- Ripples are touching the optimization community
- Specific optimization skills will be required, because optimal solution based on a nominal forecast (deterministic setting) might perform poorly under off-nominal conditions

Roger Wets’ illuminating example: deterministic vs. robust

of a furniture manufacturer deciding how many dressers of each type to make with man-hours and time as constraints; when they are uncertain, the stochastic optimal solution considers all $10^6$ possibilities and provides a robust solution, whereas the deterministic solution does not, and does not point in the right direction
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From dam to dam dynamical modelling
A single dam nonlinear dynamical model where turbinating decisions are made before knowing the water inflows

We can model the dynamics of the water volume in a dam by

\[
S(t + 1) = \min\{S^#, S(t) - q(t) + a(t)\}
\]

- **\(S(t)\)** volume (stock) of water at the beginning of period \([t, t + 1]\)
- **\(a(t)\)** inflow water volume (rain, etc.) during \([t, t + 1]\)
- decision-hazard:
  - \(a(t)\) is not available at the beginning of period \([t, t + 1]\)
- **\(q(t)\)** turbined outflow volume during \([t, t + 1]\)
  - decided at the beginning of period \([t, t + 1]\)
  - supposed to depend on the stock \(S(t)\) but not on the inflow water \(a(t)\)
  - chosen such that \(0 \leq q(t) \leq \min\{S(t), q^#\}\)
A single dam nonlinear dynamical model in decision-hazard: equivalent formulation

We can model the dynamics of the water volume in a dam by

\[
S(t+1) = S(t) - q(t) - r(t) + a(t)
\]

- **\( S(t) \)** volume (stock) of water at the beginning of period \([t, t+1]\)
- **\( a(t) \)** inflow water volume (rain, etc.) during \([t, t+1]\)
- **\( q(t) \)** turbined outflow volume during \([t, t+1]\)
  - decided at the beginning of period \([t, t+1]\)
  - supposed to depend on the stock \(S(t)\) but not on the inflow water \(a(t)\)
  - chosen such that \(0 \leq q(t) \leq \min\{S(t), q^\#\}\)
- Here, the **spilled** volume \(r(t)\) is given by the formula

\[
r(t) = \max\{0, S(t) - q(t) + a(t) - S^\#\}
\]
When turbinating decisions are made after knowing the water inflows, we obtain a linear dynamical model.

We can model the dynamics of the water volume in a dam by

\[
S(t + 1) = S(t) - q(t) - r(t) + a(t)
\]

- \(S(t)\) volume (stock) of water at the beginning of period \([t, t + 1]\)
- \(a(t)\), inflow water volume (rain, etc.) during \([t, t + 1]\);
- hazard-decision:
  - \(a(t)\) is known and available at the beginning of period \([t, t + 1]\)
  - \(q(t)\) turbined outflow volume and \(r(t)\) spilled volume
    - decided at the beginning of period \([t, t + 1]\)
    - supposed to depend on the stock \(S(t)\) and on the inflow water \(a(t)\)
    - chosen such that

\[
0 \leq q(t) \leq \min\{S(t), q^\#\} \quad \text{and} \quad 0 \leq S(t) - q(t) + a(t) - r(t) \leq S^\#
\]
Where do we stand after this brief review of single dam models?

- Information structure, nature of variables and analytical properties are interdependent
  - When turbinating decisions are made before knowing the uncertain water inflows, the spillover is an output variable (a mix of control/turbined and uncertainty/inflow variables) and the dynamical model is nonlinear
  - When turbinating decisions are made after knowing the uncertain water inflows and when the spillover is a control, the dynamical model is linear
- We now go on on our journey by exploring how we can frame dams management under uncertainty
Complexity increases with interconnected dams
Typology of hydro-valleys

(a) dams in cascade

(b) converging valleys

(c) pumping
Sketch of a cascade model with dams $i = 1, \ldots, N$

- $a_{i,t}$: inflow into dam $i$ at time $t$ (rain, run off water)
- $S_{i,t}$: volume in dam $i$ at time $t$ (water volume)
- $q_{i,t}$: turbined from dam $i$ at time $t$ (valued at price $p_{i,t}$)
- $r_{i,t}$: spilled volume from dam $i$ at time $t$ (irrigation...)

Michel DE LARA (CERMICS, France)
The dynamics describes the temporal evolution of the water stock volume in the dam.

The water volume in the dam evolves when time goes by and it is given by

\[ S_{i,t+1} = S_{i,t} + a_{i,t} + q_{i-1,t} - q_{i,t} \]

A general form is

\[ S_{i,t+1} = \text{Dyn}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, q_{i-1,t}) \]
The payoffs are decomposed in instantaneous and final

- The payoff obtained by turbining the volume $q_{i,t}$ is
  \[ p_{i,t}q_{i,t} \]

- We shall use the general form
  \[ \text{Util}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, p_{i,t}) \]

- To avoid emptying the dam at the end of the optimization process, we introduce a “water value” term bearing on the final stock volume
  \[ \text{UtilFin}_{i}(S_{i,T}) \]
The management of the dams cascade can be formulated as an intertemporal optimization problem

- The payoff attached to the dam cascade is

  \[
  \sum_{i=1}^{N} \left( \sum_{t=t_0}^{T-1} \text{Util}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, p_{i,t}) + \text{UtilFin}_{i}(S_{i,T}) \right)
  \]

- This payoff must be optimized
  - under constraints of **dynamics**

  \[
  S_{i,t+1} = \text{Dyn}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, q_{i-1,t}), \ i \in [1, N], \ t \in [t_0, T-1]
  \]

  - and under **bounds** constraints

  \[
  q_{i,t} \in [q_{i}, \bar{q}_{i}], \ i \in [1, N], \ t \in [t_0, T-1]
  \]

  \[
  S_{i,t} \in [S_{i}, \bar{S}_{i}], \ i \in [1, N], \ t \in [t_0, T]
  \]
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The deterministic optimization problem

- In the deterministic case, inflows $a_i := (a_{i,t_0}, \ldots, a_{i,T})$ and prices $p_i := (p_{i,t_0}, \ldots, p_{i,T})$ are unique and are exactly known, for every dam $i$
- With $q_i := (q_{i,t_0}, \ldots, q_{i,T-1})$ and $S_i := (S_{i,t_0}, \ldots, S_{i,T})$, the optimization problem is

$$\max_{(q_i,S_i)_{i=1,\ldots,N}} \sum_{i=1}^{N} \left( \sum_{t=t_0}^{T-1} \text{Util}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, p_{i,t}) + \text{UtilFin}_i(S_{i,T}) \right)$$

under constraints of dynamics

$$S_{i,t_0} \text{ given, } i \in [1, N]$$

$$S_{i,t+1} = \text{Dyn}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, q_{i-1,t}), \quad i \in [1, N], \ t \in [t_0, T-1]$$

and under bounds constraints

$$q_{i,t} \in \left[q_{i}, \overline{q}_{i}\right], \ i \in [1, N], \ t \in [t_0, T-1]$$

$$S_{i,t} \in \left[S_{i}, \overline{S}_{i}\right], \ i \in [1, N], \ t \in [t_0, T]$$
The deterministic optimization problem is well posed

- If, to make things simple, we assume all variables to be scalar, the optimization must be made with respect to all control variables \((\in \mathbb{R}^N(T-t_0))\) and to all “state” variables \((\in \mathbb{R}^N(T-t_0+1))\).

- The optimization yields for every dam trajectories \((q_{i,t_0}^*, \ldots, q_{i,T-1}^*)\) and \((S_{i,t_0}^*, \ldots, S_{i,T}^*)\) of the controls and of the “states”, as well as the optimal payoff.
Here are some characteristics of the deterministic optimization problem

- The resolution of the deterministic optimization problem can be made in the framework of mathematical programming, that is, the maximization of a function over $\mathbb{R}^n$ under equality and inequality constraints.

- The volumes $S_{i,t}$ are intermediary variables, completely determined by the choice of the $q_{i,t}$ (however, such intermediary variables may be used to compute gradients by means of an adjoint state).

- The optimization problem is often solved after formulating it as a linear problem and with softwares such as CPLEX.

- In the nonlinear case, the optimization problem can be difficult to solve, due to its large size, but it is known how to apply decomposition and coordination techniques.
Outline of the presentation

1. Long term industry-academy cooperation
   - Ecole des Ponts ParisTech–Cermics–Optimization and Systems
   - Industry partners

2. The transformation of power systems seen from an optimizer perspective
   - The transformation of power systems
   - Optimization is challenged

3. Moving from deterministic to stochastic dynamic optimization
   - Dam models
   - The deterministic optimization problem is well posed
   - In the uncertain framework, the optimization problem is not well posed
   - Ingredients for stochastic dynamic optimization problems

4. Two snapshots on ongoing research
   - Decomposition-coordination optimization methods under uncertainty
   - Risk constraints in optimization

5. A need for training and research
Moving from deterministic to stochastic dynamic optimization

In the uncertain framework, the optimization problem is not well posed

Water inflows historical scenarios
Description of variables: $a_{i,t}$

Inflow water volume

$a_{i,t}$: amount of water inflowing without any control into the dam

Inflow water volumes $a_{i,t}$ are part of the problem data, and may be
- either deterministic, in which case the values $(a_{i,t_0}, \ldots, a_{i,T})$ are unique and are exactly known, for every dam $i$
- or uncertain, and many possibilities can be considered
  - inflows chronicles may be available:
    $(a_{i,t_0}^s, \ldots, a_{i,T}^s)$ where $s$ belongs to a set $S$ of historical scenarios
  - in addition, the probability $\pi^s$ of scenario $s$ may be known
  - more generally, the sequence $(a_{i,t_0}, \ldots, a_{i,T})$ may be considered as a random process with known probability distribution

Prices $p_{i,t}$ can also be part of the problem data

In the same way, the prices of turbined volumes can be part of the problem data, with an uncertain or deterministic status
Description of variables: $q_{i,t}$

Turbined water volume

$q_{i,t}$: amount of water turbined to produce electricity

- Turbined volumes $q_{i,t}$ are control variables, the possible values of which are in the hands of the decision-maker to achieve different goals.
- In the deterministic framework, one looks for a single sequence of values $(q_{i,t_0}, \ldots, q_{i,T-1})$ for each dam $i$.
- In the uncertain framework, the situation is more complex:
  - one has to specify the online available information upon which decisions are made.
  - the controls $q_{i,t}$ are also uncertain variables.
  - and they can be obtained by means of feedback policies feeding on online information.
Description of variables: $r_{i,t}$

**Spilled water volume**

$r_{i,t}$: amount of water spilled from the dam with or without control

The status of the spilled volumes depends on the context, and $r_{i,t}$ can represent:

- an amount taken in the dam (for irrigation purposes, for instance), and can be a data (deterministic or uncertain)
- a volume deliberately taken in the dam and released without being turbined, and it is then a control variable
- the amount of water which overflows in case of excess

$$r_{i,t} = [S_{i,t} + a_{i,t} + q_{i-1,t} - q_{i,t} - S_i^+]$$

and it is then an output variable which results from a calculation based upon other variables
In the uncertain framework, two additional questions must be answered with respect to the deterministic case.

Question (expliciting risk attitudes)
How are the uncertainties taken into account in the payoff criterion and in the constraints?

Question (expliciting available online information)
Upon which online information are turbining decisions made?

In practice, one assumes
- more or less precise knowledge of the past (online information)
- statistical assumptions about the future (offline information)
How are the uncertainties taken into account in the payoff criterion and in the constraints?

In a probabilistic setting, where uncertainties are random variables, a classical answer is

- to take the mathematical expectation of the payoff (risk-neutral approach)

\[
\mathbb{E}\left( \sum_{i=1}^{N} \left( \sum_{t=t_0}^{T-1} \text{Util}_{i,t}(S_{i,t}, q_{i,t}, a_{i,t}, p_{i,t}) + \text{UtilFin}_{i}(S_{i,T}) \right) \right)
\]

- and to satisfy all (physical) constraints almost surely that is, practically, for all possible issues of the uncertainties (robust approach)

\[
P\left( q_{i,t} \in \left[ q_{i}, \bar{q}_{i} \right] , \ i \in [1, N], \ t \in [t_0, T-1] \right. \bigg| \left. S_{i,t} \in \left[ S_{i}, \bar{S}_{i} \right] , \ i \in [1, N], \ t \in [t_0, T] \right) = 1
\]
Upon which online information are turbinating decisions made?

- On the one hand, it is suboptimal to restrict oneself, as in the deterministic case, to open-loop controls depending only upon time.
- On the other hand, it is impossible to suppose that we know in advance what will happen for all times: clairvoyance is impossible.

The in-between is non anticipativity constraint.

In our case, the decision $q_{i,t}$ will be looked after as a

- fonction of past uncertainties,
- that is, of the water inflows and the prices $(a_{j,\tau}, p_{j,\tau})$ for $j \in [1, N]$ and $\tau \in [t_0, t]$. 
On-line information feeds turbinating decisions: the non anticipativity constraint

Denote the uncertainties at time $t$ by

$$ w_t = (a_{1,t}, p_{1,t}, \ldots, a_{N,t}, p_{N,t}) $$

- **Functional approach**
  The control $q_{i,t}$ may be looked after under the form

  $$ q_{i,t} = \phi_{i,t}(w_{t_0}, \ldots, w_{t-1}) $$

  where $\phi_{i,t}$ is a function, called policy, strategy or decision rule

- **Algebraic approach**
  When uncertainties are considered as random variables (measurable mappings), the above formula for $q_{i,t}$ expresses the measurability of the control variable $q_{i,t}$ with respect to the past uncertainties, also written as

  $$ \sigma(q_{i,t}) \subseteq \sigma(w_{t_0}, \ldots, w_{t-1}) \iff q_{i,t} \preceq \sigma(w_{t_0}, \ldots, w_{t-1}) $$
There are two ways to express the information constraints

- **Functional approach** \( q_{i,t} = \phi_{i,t}(w_{t_0}, \ldots, w_{t-1}) \)

  In the special case of the Markovian framework with white noise, it can be shown that there is no loss of optimality to look for solutions under the form

  \[
  q_{i,t} = \psi_{i,t}(S_1,t, \ldots, S_N,t) \]

- **Measurability constraints** \( q_{i,t} \leq \sigma(w_{t_0}, \ldots, w_{t-1}) \)

  In the special case of the scenario tree approach, uncertainties are supposed to be observed, and all possible scenarios \( (w^s_{t_0}, \ldots, w^s_T), s \in S \), are organized in a tree, and controls \( q_{i,t} \) are indexed by nodes on the tree.

  Equivalently, \( q_{i,t} \) are indexed by \( s \in S \) with the constraint that if two scenarios coincide up to time \( t \), so must do the controls at time \( t \)

  \[
  (w^s_{t_0}, \ldots, w^s_{t-1}) = (w'^s_{t_0}, \ldots, w'^s_{t-1}) \Rightarrow q^s_i = q'^s_i
  \]
On-line information structure can reflect decentralized information

When the control $q_{i,t}$ is looked after under the form

$$q_{i,t} = \phi_{i,t} \left( a_{i,t_0}, p_{i,t_0}, \ldots, a_{i,T}, p_{i,T}, \ldots, a_{i,t-1}, p_{i,t-1} \right)$$

this expresses that each dam is managed with local information.
The stochastic optimization problem

In a probabilistic setting, where uncertainties are random variables and where a probability \( P \) is given, a possible formulation is

\[
\max_{(q_i, S_i)_{i=1,...,N}} \mathbb{E}\left( \sum_{i=1}^{N} \left( \sum_{t=t_0}^{T-1} \text{Util}_{i, t}(S_{i, t}, q_{i, t}, a_{i, t}, p_{i, t}) + \text{UtilFin}_i(S_{i, T}) \right) \right)
\]

- under constraints of dynamics
  
  \[
  S_{i, t_0} \text{ given, } \quad i \in \llbracket 1, N \rrbracket
  \]
  
  \[
  S_{i, t+1} = \text{Dyn}_{i, t}(S_{i, t}, q_{i, t}, a_{i, t}, q_{i-1, t}), \quad i \in \llbracket 1, N \rrbracket, \ t \in \llbracket t_0, T-1 \rrbracket
  \]

- under bounds constraints
  
  \[
  \mathbb{P}\left( q_{i, t} \in [q_{ij}, \overline{q}_{ij}], \quad \overline{S}_{i, t} \in [\underline{S}_i, \overline{S}_i], \quad i \in \llbracket 1, N \rrbracket, \ t \in \llbracket t_0, T \rrbracket \right) = 1
  \]

- and under measurability constraints
  
  \[
  q_{i, t} \preceq \sigma(w_{t_0}, \ldots, w_{t-1}), \quad i \in \llbracket 1, N \rrbracket, \ t \in \llbracket t_0, T-1 \rrbracket
  \]
Moving from deterministic to stochastic dynamic optimization

The outputs of a stochastic optimization problem are random variables
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5. A need for training and research
Let us start by lining up the ingredients for a deterministic sequential optimization problem.

- A set \( \{t_0, t_0 + 1, \ldots, T\} \subset \mathbb{N} \) of discrete times \( t \)
- Control sets \( \mathbb{U}_t \) containing control variable \( u_t \in \mathbb{U}_t \), for \( t = t_0, t_0 + 1, \ldots, T \)
- A criterion \( j(u_{t_0}, \ldots, u_T) \)
- Constraints of the form \( u_t \in \mathbb{U}_t' \subset \mathbb{U}_t \)

**Two-stage problem**

Times \( t \in \{0, 1\} \), \( \mathbb{U}_0 = \mathbb{U}_1 = \mathbb{R} \) and criterion \( L_0(u_0) + L_1(u_1) \)
Let us line up the ingredients for a stochastic sequential optimization problem

- A set \( \{t_0, t_0 + 1, \ldots, T\} \subset \mathbb{N} \) of discrete times \( t \)
- Control sets \( U_t \) containing control variable \( u_t \in U_t \), for \( t = t_0, t_0 + 1, \ldots, T \)
- Constraints of the form \( u_t \in U'_t \subset U_t \)
- A set \( \Omega \) of scenarios, or states of Nature, with generic element \( \omega \) (without temporal structure, a priori)
- A criterion \( j(u_{t_0}, \ldots, u_T, \omega) \)

**Two-stage problem**

Times \( t \in \{0, 1\} \), \( U_0 = U_1 = \mathbb{R} \) and criterion \( L_0(u_0) + L_1(u_1, \omega) \)

- **Stochastic optimization** highlights risk attitudes tackling
- **Stochastic dynamic** optimization emphasizes the handling of online information
For the purpose of handling online information, we introduce fields and subfields

1. \((\Omega, \mathcal{A})\) a measurable space (uncertainties, states of Nature)
2. \((\mathcal{U}_{t_0}, \mathcal{U}_{t_0}), \ldots, (\mathcal{U}_T, \mathcal{U}_T)\) measurable spaces (decision spaces)
3. Subfield \(I_t \subset \mathcal{U}_{t_0} \otimes \cdots \otimes \mathcal{U}_{t-1} \otimes \mathcal{A}\), for \(t = t_0, \ldots, T\) (information)

The inclusion

\[ I_t \subset \mathcal{U}_{t_0} \otimes \cdots \otimes \mathcal{U}_{t-1} \otimes \mathcal{A} \]

captures the fact that the information at time \(t\) is made at most of past controls and the state of Nature (causality)
The interplay between decision and information is a distinguishing trait of stochastic dynamic optimization compared to the deterministic setting

Decision rule, policy, strategy

A strategy is a sequence $\lambda = \{\lambda_t\}_{t=t_0,\ldots,T}$ of measurable mappings from past histories to decision sets

$$
\lambda_{t_0} : (\Omega, \mathcal{A}) \rightarrow (\mathcal{U}_{t_0}, \mathcal{U}_{t_0}) \\
\lambda_t : (\mathcal{U}_{t_0} \times \cdots \times \mathcal{U}_{t-1} \times \Omega, \mathcal{U}_{t_0} \otimes \cdots \otimes \mathcal{U}_{t-1} \otimes \mathcal{A}) \rightarrow (\mathcal{U}_t, \mathcal{U}_t)
$$

satisfying the information constraints for $t = t_0, \ldots, T$

$$
\lambda_t^{-1}(\mathcal{U}_t) \subseteq \mathcal{I}_t
$$

information
The solution map is attached to a policy, and maps a scenario towards a history

**Solution map**

With a policy $\lambda$, we associate the mapping

$$S_\lambda : \Omega \rightarrow \mathbb{U}_{t_0} \times \cdots \times \mathbb{U}_T \times \Omega$$

defined by

$$(u_{t_0}, \ldots, u_T, \omega) = S_\lambda(\omega) \iff \begin{cases} u_{t_0} = \lambda_{t_0}(\omega) \\ u_{t_0+1} = \lambda_{t_0+1}(u_{t_0}, \omega) \\ \vdots \\ u_T = \lambda_T(u_{t_0}, \ldots, u_{T-1}, \omega) \end{cases}$$
Nonanticipativity constraint

- Product scenario space

\[ \Omega = \prod_{t=t_0+1}^{T} W_t \text{ with } \mathcal{A} = \bigotimes_{t=t_0+1}^{T} W_t \]

- Past uncertainties fields for \( t = t_0 + 1, \ldots, T \),

\[ \mathcal{A}_t = W_{t_0+1} \otimes \cdots \otimes W_t \otimes \{\emptyset, W_{t+1}\} \otimes \cdots \otimes \{\emptyset, W_T\} \]

- Nonanticipativity constraint

\[ \mathcal{I}_{t_0} = \{\emptyset, \Omega\} \text{ and } \mathcal{I}_t \subset U_{t_0} \otimes \cdots \otimes U_{t-1} \otimes \mathcal{A}_t \]
For the purpose of handling risk attitudes, we introduce a probability or any risk measure.

With a policy $\lambda$, we associate the function $j \circ S_\lambda : \Omega \to \mathbb{R}$

Stochastic optimization problem

When $(\Omega, \mathcal{A}, \mathbb{P})$ is a probability space, the stochastic optimization problem is

$$\min_{\lambda_{t_0} \leq \mathcal{I}_{t_0}, \ldots, \lambda_T \leq \mathcal{I}_T} \mathbb{E}\left( j(S_\lambda(\cdot)) \right)$$

where $\lambda_t \leq \mathcal{I}_t$ means that $\lambda_t$ is $\mathcal{I}_t$-measurable, that is, $\lambda_t^{-1}(\mathcal{U}_t) \subset \mathcal{I}_t$, for $t = t_0, \ldots, T$

When $\bar{\Omega} \subset \Omega$, the robust optimization problem is

$$\min_{\lambda_{t_0} \leq \mathcal{I}_{t_0}, \ldots, \lambda_T \leq \mathcal{I}_T} \sup_{\omega \in \bar{\Omega}} j(S_\lambda(\omega))$$
Two-stage stochastic programming problem

\[
\min_{u_0} L_0(u_0) + \mathbb{E} \left( \min_{u_1} L_1(u_1, \omega_1) \right)
\]

- **Decision spaces**
  \[(U_0, U_0) = (\mathbb{R}^{p_0}, \mathcal{B}^o_{\mathbb{R}^{p_0}}) \text{ and } (U_1, U_1) = (\mathbb{R}^{p_1}, \mathcal{B}^o_{\mathbb{R}^{p_1}})\]

- **Probability** \(\mathbb{P}\) on the probability space
  \[\Omega = \mathcal{W}_1 = \mathbb{R}^{q_1} \text{ with } \mathcal{A} = \mathcal{B}^o_{\mathcal{W}_1} = \mathcal{B}^o_{\mathbb{R}^{q_1}}\]

- **Information fields**
  \[I_0 = \{\emptyset, \Omega\} \text{ and } I_1 = U_0 \otimes \mathcal{A}\]

- at the first stage, there is no information whatsoever
- at the second stage, the first decision and the state of Nature are available for decision-making
Multi-stage stochastic programming problem

\[
\min_{u_{t_0}} L_{t_0}(u_{t_0}) + 
\mathbb{E}\left( \min_{u_{t_0+1}} L_{t_0+1}(u_{t_0+1}, \omega_{t_0+1}) + \mathbb{E}\left( \cdots + \mathbb{E}\left( \min_{u_T} L_T(u_T, \omega_T) \right) \right) \right),
\]

This corresponds to the decision spaces

\[(U_{t_0}, U_{t_0}) = (\mathbb{R}^{p_{t_0}}, \mathcal{B}_{\mathbb{R}^{p_{t_0}}}), \ldots, (U_T, U_T) = (\mathbb{R}^{p_T}, \mathcal{B}_{\mathbb{R}^{p_T}}),\]

and to the probability space

\[\Omega = \prod_{t=t_0+1}^{T} \mathcal{W}_t \quad \text{with} \quad \mathcal{A} = \bigotimes_{t=t_0+1}^{T} \mathcal{W}_t\]

equipped with a probability \(\mathbb{P}\)
State model and dynamic programming (DP)

- Dynamics with an intermediary variable $x_t \in X_t$
  \[ x_{t+1} = f_t(x_t, u_t, w_t), \quad t = t_0, \ldots, T \]

- Criterion $j(x(\cdot), u(\cdot), w(\cdot))$

- White noise assumption: the scenario space $\Omega = \prod_{t=t_0+1}^{T} W_t$ is equipped with a product probability
  \[ \mathbb{P} = \bigotimes_{t=t_0+1}^{T} \mu_t \]

- Then $x_t \in X_t$ deserves the name of state: $x_t$ summarizes the past history in that, given time $t$ and the value of $x_t$, one can calculate the optimal $u_t$ and also $x_{t+1}$ without knowledge of the whole history $(u_{t_0}, \ldots, u_{t-1}, \omega)$, for all $t$
We distinguish classes of stochastic optimization problems

- Assumptions on the scenario set $\Omega$
  - DP: product set
  - SP: finite product set

- Assumptions on the probability $\mathbb{P}$
  - DP: product probability (enables a reduction of past history into a state)
  - SP: no specific assumption (apart from discrete distribution)

- Analytical assumptions on the criterion $j$
  - DP: no specific assumption
  - SP: linearity, convexity

Solving a sequence of two-step optimization problems indeed amounts to solving an intertemporal optimization problem when some form of dynamic consistency holds true.
Optimization approaches to attack complexity

Linear programming
- linear equations and inequalities
- no curse of dimensionality

Stochastic programming
- no special treatment of time and uncertainties
- no independence assumption
- decisions are indexed by a scenario tree
- what if information is not a node in the tree?

State-based dynamic optimization
- nonlinear equations and inequalities
- curse of dimensionality
- independence assumption on uncertainties
- special treatment of time (dynamic programming equation)
- decisions are indexed by an information state (feedback synthesis)
- an information state summarizes past controls and uncertainties
- decomposition-coordination methods to overcome the curse of dimensionality?
Summary

- Stochastic optimization highlights risk attitudes tackling
- Stochastic dynamic optimization emphasizes the handling of online information
- Many open problems, because
  - many ways to represent risk (criterion, constraints)
  - many information structures
  - tremendous numerical obstacles to overcome
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3. Moving from deterministic to stochastic dynamic optimization

4. Two snapshots on ongoing research

5. A need for training and research
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Decomposition-coordination: divide and conquer

- **Spatial** decomposition
  - multiple players with their local information
  - local / regional / national / supranational

- **Temporal** decomposition
  - A state is an information summary
  - Time coordination realized through Dynamic Programming, by value functions
  - Hard nonanticipativity constraints

- **Scenario** decomposition
  - Along each scenario, sub-problems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints
Coupling constraints: an overview

\[
\min_{x, u} \sum_{\omega} \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i, t}(x_{i, t}, u_{i, t}, w_t)
\]
Coupling constraints: time coupling

\[
\begin{align*}
\min_{x,u} & \quad \sum_{\omega} \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_t) \\
\text{s.t.} & \quad x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_t)
\end{align*}
\]
Coupling constraints: scenario coupling

\[
\min_{x, u} \sum_{\omega} \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_t)
\]

s.t. \( x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_t) \)

\[ \sigma(u_{i,t}) \subset \sigma(w_0, \ldots, w_t) \]
Coupling constraints: space coupling

\[
\begin{align*}
\min_{x, u} & \quad \sum_{\omega} \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_{t}) \\
\text{s.t.} & \quad x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_{t}) \\
& \quad \sigma(u_{i,t}) \subset \sigma(w_{0}, \ldots, w_{t}) \\
& \quad \sum_{i=1}^{N} \theta_{i,t}(x_{i,t}, u_{i,t}, w_{t}) = 0
\end{align*}
\]
Decomposition/coordination methods: an overview

Main idea

1. **decompose** a large scale problem into smaller subproblems we are able to solve by efficient algorithms
2. **coordinate** the subproblems for the concatenation of their solutions to form the initial problem solution

How to decompose the problem by duality?

1. **identify** the coupling dimensions of the problem: time, uncertainty, space
2. **dualize** the coupling constraints by introducing **multiplyers**
3. **split** the problem into the resulting subproblems and **coordinate** them by means of the multiplier

In the case of **time decomposition**, we can use the time arrow to **chain** static subproblems by the dynamics equation (without dualizing)
Decomposition/coordination methods: an overview

\[
\min_{x, u} \mathbb{E}\left( \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_t) \right) \\
\text{s.t. } x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_t) \\
\sigma(u_{i,t}) \subset \sigma(w_0, \ldots, w_t) \\
\sum_{i=1}^{N} \theta_{i,t}(x_{i,t}, u_{i,t}, w_t) = 0
\]
Decomposition/coordination methods: time coupling

\[
\min_{x,u} \mathbb{E} \left( \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_t) \right)
\]

s.t. \[x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_t)\]
\[\sigma(u_{i,t}) \subset \sigma(w_0, \ldots, w_t)\]
\[\sum_{i=1}^{N} \theta_{i,t}(x_{i,t}, u_{i,t}, w_t) = 0\]

[Stochastic Pontryagin]
[Dynamic Programming]
Decomposition/coordination methods: scenario coupling

\[
\min_{x, u} \mathbb{E} \left( \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_t) \right)
\]

s.t. \( x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_t) \)

\( \sigma(u_{i,t}) \subseteq \sigma(w_0, \ldots, w_t) \)

\[
\sum_{i=1}^{N} \theta_{i,t}(x_{i,t}, u_{i,t}, w_t) = 0
\]

[Progressive Hedging]

Decomposition/coordination methods: space coupling

\[ \min_{x, u} \mathbb{E} \left( \sum_{i=1}^{N} \sum_{t=0}^{T} L_{i,t}(x_{i,t}, u_{i,t}, w_t) \right) \]

s.t. \[ x_{i,t+1} = f_{i,t}(x_{i,t}, u_{i,t}, w_t) \]

\[ \sigma(u_{i,t}) \subset \sigma(w_0, \ldots, w_t) \]

\[ \sum_{i=1}^{N} \theta_{i,t}(x_{i,t}, u_{i,t}, w_t) = 0 \]

[Our purpose now]
We have a nice decomposed problem but...  

Flower structure

We are almost in the case where units could be driven independently one from another
We have a nice decomposed problem but...  

Flower structure

Unfortunately...
The associated optimization problem may be written as

\[
\min_{(u_1, \ldots, u_N)} \sum_{i=1}^{N} J_i(u_i) \quad \text{under} \quad \sum_{i=1}^{N} \Theta_i(u_i) = D
\]

where

- \( u_i \) is the decision of each unit \( i \)
- \( J_i(u_i) \) is the cost of making decision \( u_i \) for unit \( i \)
- \( \Theta_i(u_i) \) is the production induced by making decision \( u_i \) for unit \( i \)
Under appropriate duality assumptions, the associated optimization problem can be written without constraints

- For a proper Lagrange multiplier $\lambda$

$$
\min_{(u_1,\ldots,u_N)} \sum_{i=1}^{N} J_i(u_i) + \lambda \left( \sum_{i=1}^{N} \Theta_i(u_i) - D \right)
$$

- We distribute the coupling constraint to each unit $i$

$$
\min_{(u_1,\ldots,u_N)} \left( \sum_{i=1}^{N} J_i(u_i) + \lambda \Theta_i(u_i) \right) - \lambda D
$$

- The problems splits into $N$ optimization problems

$$
\min_{u_i} \left( J_i(u_i) + \lambda \Theta_i(u_i) \right), \quad \forall i = 1, \ldots, N
$$
Proper prices allow decentralization of the optimum

\[
\min_{(u_1, \ldots, u_N)} \sum_{i=1}^{N} J_i(u_i) \quad \text{under} \quad \sum_{i=1}^{N} \Theta_i(u_i) = D
\]

The simplest decomposition/coordination scheme consists in

- buying the production of each unit at a price \( \lambda^{(k)} \)
- and letting each unit minimizing its costs

\[
\min_{u_i} J_i(u_i) + \lambda^{(k)} \Theta_i(u_i)
\]

then, modifying the price depending on the coupling constraint

\[
\lambda^{(k+1)} = \lambda^{(k)} + \rho \left( \sum_{i=1}^{N} \Theta_i(u_i) - D \right)
\]

(like in the “tâtonnement de Walras” in economics)
What are the stakes if we extend spatial coupling constraint decomposition to the dynamical and stochastic setting?

If we explicitly take into account time and uncertainties, a classical approach consists in writing the new problem

$$\min_{\{u_i,t\}_{i \in \{1,N\}}, \{t\in\{0,T-1\}}} \mathbb{E} \left( \sum_{i=1}^{N} \left( \sum_{t=0}^{T-1} L_i,t(x_i,t, u_i,t, w_i,t) + K_i(x_i,T) \right) \right)$$

under the constraints

$$\sum_{i=1}^{N} \Theta_{i,t}(x_i,t, u_i,t, w_i,t) - d_t = 0, \quad t \in [0, T-1]$$

$$x_{i,t+1} = f_{i,t}(x_i,t, u_i,t, w_i,t), \quad i \in [1, N], \quad t \in [0, T-1]$$
We need to specify information constraints

- The optimization problem is not well posed, because we have not specified upon what depends the control $u_{i,t}$ of each unit $i$ at each time $t$.

- In the causal and perfect memory case, we express that the control $u_{i,t}$ depends on all past noises up to time $t$.
  - either by a functional approach
    \[ u_{i,t} = \phi_{i,t}(w_{1,0}, \ldots, w_{N,0}, d_0 \ldots \ldots w_{1,t}, \ldots, w_{N,t}, d_t) \]
  - or by an algebraic approach
    \[ \sigma(u_{i,t}) \subset \sigma(w_{1,0}, \ldots, w_{N,0}, d_0 \ldots \ldots w_{1,t}, \ldots, w_{N,t}, d_t) \]

When the control $u_{i,t}$ is looked after under the form
\[ u_{i,t} = \phi_{i,t}(w_{i,0}, d_0 \ldots \ldots w_{i,t}, d_t) \] or satisfying
\[ \sigma(u_{i,t}) \subset \sigma(w_{i,0}, d_0 \ldots \ldots w_{i,t}, d_t) \], this expresses that each unit is managed with local information, which is a way to handle decentralized information.
Looking after decentralizing prices models

- Going on with the previous scheme, each unit $i$ solves

$$
\min_{u_{i,0}, \ldots, u_{i,T-1}} \mathbb{E} \left( \sum_{t=0}^{T-1} \left( \mathbf{L}_{i,t}(\mathbf{x}_{i,t}, \mathbf{u}_{i,t}, \mathbf{w}_{i,t}) + \mathbf{\lambda}_{i,t}^{(k)} \mathbf{\Theta}_{i,t}(\mathbf{x}_{i,t}, \mathbf{u}_{i,t}, \mathbf{w}_{i,t}) \right) + K_i(\mathbf{x}_{i,T}) \right)
$$

under the constraints

$$
\mathbf{x}_{i,t+1} = f_{i,t}(\mathbf{x}_{i,t}, \mathbf{u}_{i,t}, \mathbf{w}_{i,t}), \quad t \in [0, T-1]
$$

- In all rigor, the optimal controls $\mathbf{u}_{i,t}^*$ of this problem depend
  - upon the local state $\mathbf{x}_{i,t}$
  - and ... upon all past prices ($\mathbf{\lambda}_{i,0}^{(k)}, \ldots, \mathbf{\lambda}_{i,t}^{(k)}$)!

  **Research axis:** find an approximate dynamical model for the prices, driven by proper information; for instance, replace $\mathbf{\lambda}_{i,t}^{(k)}$ by $\mathbb{E} \left( \mathbf{\lambda}_{i,t}^{(k)} \mid y_t \right)$, where the information variable $y_t$ is a short time memory process.
Dual Approximate Dynamic Programming

Samples/scenarios of
dual variable
at iteration $k$

$\lambda^k$
Two snapshots on ongoing research
Decomposition-coordination optimization methods under uncertainty

Dual Approximate Dynamic Programming

Samples/scenarios of
dual variable
at iteration $k$

We solve subproblems
using $\mathbb{E}(\lambda^k | y)$
by Dynamic Programming
Dual Approximate Dynamic Programming

Samples/scenarios of dual variable at iteration $k$

We solve subproblems using $\mathbb{E}(\lambda^k | y)$ by Dynamic Programming

We obtain policies

\[ \lambda^k \]

Subproblem 1

Subproblem $i$ (for $i = 1, 2, ..., N$)

Policy $\Phi^1(x^1, y)$

Policy $\Phi^i(x^i, y)$

Policy $\Phi^N(x^N, y)$
Two snapshots on ongoing research

Decomposition-coordination optimization methods under uncertainty

Dual Approximate Dynamic Programming

Samples/scenarios of dual variable at iteration $k$.

We solve subproblems using $\mathbb{E}(\lambda^k | y)$ by Dynamic Programming.

We obtain policies.

We update prices using a gradient step.

We simulate scenarios of strategies and prices and update prices using a gradient step.

\[ \lambda_{t+1}^k = \lambda_t^k + \rho \times \sum_{i=1}^{N} g_t^i (x_{t}^{i,k}, u_{t}^{i,k}, w_t) \]
Dual Approximate Dynamic Programming

At iteration $k + 1$

We solve subproblems using $\mathbb{E}(\lambda^k | y)$ by Dynamic Programming

We obtain policies

We update prices using a gradient step

We simulate scenarios of strategies and prices and update prices using a gradient step

$$\lambda^{k+1} = \lambda^k + \rho \times \sum_{i=1}^{N} g_i^t (x^{i,k}, u^{i,k}, w_t)$$
Extension to interconnected dams
Contribution to dynamic tariffs

- Spatial decomposition of a dynamic stochastic optimization problem
- Lagrange multipliers attached to spatial coupling constraints are stochastic processes (prices)
- By projecting these prices, one expects to identify approximate dynamic models
- Such prices dynamic models are interpreted as dynamic tariffs
Outline of the presentation

1. Long term industry-academy cooperation
   - Ecole des Ponts ParisTech–Cermics–Optimization and Systems
   - Industry partners

2. The transformation of power systems seen from an optimizer perspective
   - The transformation of power systems
   - Optimization is challenged

3. Moving from deterministic to stochastic dynamic optimization
   - Dam models
   - The deterministic optimization problem is well posed
   - In the uncertain framework, the optimization problem is not well posed
   - Ingredients for stochastic dynamic optimization problems

4. Two snapshots on ongoing research
   - Decomposition-coordination optimization methods under uncertainty
   - Risk constraints in optimization

5. A need for training and research
A hydro-electric dam has to be managed under a “tourism” constraint

- Maximizing the revenue from turbinated water
- under a tourism constraint of having enough water in July and August
The red stock trajectories fail to meet the tourism constraint in July and August.
We consider a single dam nonlinear dynamical model in the decision-hazard setting

We can model the dynamics of the water volume in a dam by

\[
S(t + 1) = \min\{S^\#, S(t) - q(t) + a(t)\}
\]

- **\(S(t)\)** volume (stock) of water at the beginning of period \([t, t + 1[\]
- **\(a(t)\)** inflow water volume (rain, etc.) during \([t, t + 1[\]
- decision-hazard:
  - \(a(t)\) is not available at the beginning of period \([t, t + 1[\]
- **\(q(t)\)** turbined outflow volume during \([t, t + 1[\]
  - decided at the beginning of period \([t, t + 1[\]
  - supposed to depend on \(S(t)\) but not on \(a(t)\)
  - chosen such that \(0 \leq q(t) \leq \min\{S(t), q^\#\}\)
In the risk-neutral economic approach, an optimal management maximizes the expected payoff

- Suppose that
  - a probability \( \mathbb{P} \) is given on the set \( \Omega = \mathbb{R}^{T-t_0} \) of water inflows scenarios \( (a(t_0), \ldots, a(T-1)) \)
  - turbined water \( q(t) \) is sold at price \( p(t) \), related to the price at which energy can be sold at time \( t \)
  - at the horizon, the final volume \( S(T) \) has a value \( K(S(T)) \), the “final value of water”

- The traditional (risk-neutral) economic problem is to maximize the intertemporal payoff (without discounting if the horizon is short)

\[
\max \mathbb{E} \left[ \sum_{t=t_0}^{T-1} p(t)q(t) + K(S(T)) \right]
\]
We now have a stochastic optimization problem, where the “tourism” constraint still needs to be dressed in formal clothes

- Traditional cost minimization/payoff maximization

\[
\max \mathbb{E} \left[ \sum_{t=t_0}^{T-1} \left( p(t)q(t) \right) + K(S(T)) \right]
\]

- For “tourism” reasons:

\[
\text{volume } S(t) \geq S^b, \quad \forall t \in \{ \text{July, August} \}
\]

- In what sense should we consider this inequality which involves the random variables \( S(t) \) for \( t \in \{ \text{July, August} \} \)?
Robust / almost sure / probability constraint

- **Robust constraints**: for all the scenarios in a subset $\Omega \subset \Omega$
  \[ S(t) \geq S^b, \quad \forall t \in \{ \text{July, August} \} \]

- **Almost sure constraints**

  \[
  \text{Probability} \left\{ \begin{array}{c}
  \text{water inflow scenarios along which the volumes } S(t) \text{ are above the threshold } S^b \\
  \text{for periods } t \text{ in summer}
  \end{array} \right\} = 1
  \]

- **Probability constraints**, with “confidence” level $p \in [0, 1]$

  \[
  \text{Probability} \left\{ \begin{array}{c}
  \text{water inflow scenarios along which the volumes } S(t) \text{ are above the threshold } S^b \\
  \text{for periods } t \text{ in summer}
  \end{array} \right\} \geq p
  \]

- and also by penalization, or in the mean, etc.
Our problem may be clothed as a stochastic optimization problem under a probability constraint

- The traditional economic problem is \( \max \mathbb{E}[P(T)] \)
  where the payoff/utility criterion is

\[
P(T) = \sum_{t=t_0}^{T-1} \text{turbined water payoff } p(t)q(t) + \text{final volume utility } K(S(T))
\]

- and a failure tolerance is accepted

\[
\text{Probability } \left\{ \begin{array}{l}
\text{water inflow scenarios along which } S(t) \geq S^p \\
\text{for periods } t \text{ in July and August}
\end{array} \right\} \geq 99\%
\]

- Details concerning the theoretical and numerical resolution are available on demand ;-)
90% of the stock trajectories meet the tourism constraint in July and August
Our resolution approach brings a sensible improvement compared to standard procedures.

<table>
<thead>
<tr>
<th>OPTIMAL POLICIES</th>
<th>OPTIMIZATION</th>
<th>SIMULATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Iterations</td>
<td>Time</td>
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<tr>
<td>Standard</td>
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<td>10 mn</td>
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<tr>
<td>Convenient</td>
<td>10</td>
<td>160 mn</td>
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<tr>
<td>Heuristic</td>
<td>10</td>
<td>160 mn</td>
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</tbody>
</table>
However, though the expected payoff is optimal, the payoff effectively realized can be far from it.
We propose a stochastic viability formulation to treat symmetrically and to guarantee both environmental and economic objectives.

- Given two thresholds to be guaranteed:
  - a volume $S^b$ (measured in cubic hectometers $hm^3$)
  - a payoff $P^b$ (measured in numeraire $\$$)

we look after policies achieving the maximal viability probability

$$\Pi(S^b, P^b) = \max \text{ Proba } \left\{ \begin{array}{l}
\text{water inflow scenarios along which the volumes } S(t) \geq S^b \\
\text{for all time } t \in \{ \text{July, August} \} \\
\text{and the final payoff } P(T) \geq P^b
\end{array} \right\}$$

- The maximal viability probability $\Pi(S^b, P^b)$ is the maximal probability to guarantee to be above the thresholds $S^b$ and $P^b$
The stochastic viability formulation requires to redefine state and dynamics

- The state is the couple \( x(t) = (S(t), P(t)) \) volume/payoff
- The control \( u(t) = q(t) \) is the turbined water
- The dynamics is

\[
\begin{align*}
S(t + 1) &= \min\{S^\#, S(t) - q(t) + a(t)\}, \\
& \quad \text{future volume volume turbined inflow volume} \\
& \quad t = t_0, \ldots, T - 1 \\

P(t + 1) &= P(t) + p(t)q(t), \\
& \quad \text{future payoff payoff turbined water payoff} \\
& \quad t = t_0, \ldots, T - 2 \\

P(T) &= P(T - 1) + K(S(T)), \\
& \quad \text{final volume utility}
\end{align*}
\]
In the stochastic viability formulation, objectives are formulated as state constraints

- The control constraints are

\[ u(t) \in \mathbb{B}(t, x(t)) \iff 0 \leq q(t) \leq S(t) \]

- The state constraints are

\[ x(t) \in A(t) \iff \begin{cases} S(t) \geq S^b \\ P(T) \geq P^b \end{cases} , \quad \forall t \in \{ \text{July, August} \} \]
For each couple of thresholds on payoff and stock, we write a dynamic programming equation

- **Abstract version**
  \[
  V(T, x) = 1_{A(T)}(x)
  \]
  \[
  V(t, x) = 1_{A(t)}(x) \max_{u \in B(t, x)} \mathbb{E}_w(t) \left[ V(t + 1, \text{Dyn}(t, x, u, w(t))) \right]
  \]

- **Specific version**
  \[
  V(T, S, P) = 1_{\{ P \geq P^b \}}
  \]
  \[
  V(T - 1, S, P) = \max_{0 \leq q \leq S} \mathbb{E}_a(t) \left[ V(t + 1, S - q + a(t), P + K(S)) \right]
  \]
  \[
  V(t, S, P) = \max_{0 \leq q \leq S} \mathbb{E}_a(t) \left[ V(t + 1, S - q + a(t), P + pq) \right], \quad t \notin \{ \text{July, August} \}
  \]
  \[
  V(t, S, P) = 1_{\{ S \geq s^b \}} \max_{0 \leq q \leq S} \mathbb{E}_a(t) \left[ V(t + 1, S - q + a(t), P + pq) \right], \quad t \in \{ \text{July, August} \}
  \]
We plot the maximal viability probability $\Pi(S^b, P^b)$ as a function of guaranteed thresholds $S^b$ and $P^b$.

For example, the probability to guarantee
- a final payoff above $P^b = 1$ Meuros
- and a volume above $S^b = 40$ hm$^3$ in July and August

is about 90%
We plot iso-values for the maximal viability probability as a function of guaranteed thresholds $S^b$ and $P^b$.
Contribution to quantitative sustainable management

- Conceptual framework for quantitative sustainable management
- Managing ecological and economic conflicting objectives
- Displaying tradeoffs between ecology and economy sustainability thresholds and risk
Outline of the presentation

1. Long term industry-academy cooperation
2. The transformation of power systems seen from an optimizer perspective
3. Moving from deterministic to stochastic dynamic optimization
4. Two snapshots on ongoing research
5. A need for training and research
A need for training and research

Trends are favorable to statistics and optimization

- More telecom technology $\rightarrow$ more data
- More data, more unpredictability $\rightarrow$ more statistics
- More unpredictability $\rightarrow$ more storage $\rightarrow$ more dynamic optimization
- More unpredictability $\rightarrow$ more stochastic dynamic optimization
A context of increasing complexity

- Multiple energy resources: photovoltaic, solar heating, heatpumps, wind, hydraulic power, combined heat and power
- Spatially distributed energy resources (onshore and offshore windpower, solarfarms), producers, consumers
- Strongly variable production: wind, solar
- Intermittent demand: electrical vehicles
- Two-ways flows in the electrical network
- Environmental and risk constraints (CO2, nuclear risk, land use)
Challenges ahead for stochastic optimization

- large scale stochastic optimization
- various risk constraints
- decentralized and private information
- game theory, stochastic equilibrium, market design, etc.