

Report on Junior Hausdorff Semester Program “Computational Mathematics” Extreme Geometric Structures

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Extreme and sporadic geometrical structures are at the core of many mathematical theories. In the last few years, computers were heavily used to study such structures. Tools from optimization were used to determine locally or globally ones. With efficient implementations of combinatorial algorithms classification results were established.

During the three months in Bonn we concentrated on extreme structures coming from special distributions of points in different geometric spaces. We focused on point configurations on Euclidean spheres and Euclidean space which are good with respect to parameters like packing density, covering density, minimal energy, or regularity.

These problems are connected to many areas in mathematics, mathematical physics and material science. When studying extreme structures there are two basic tasks on which we worked: The construction of (conjectural) extreme structures and on proofs that given structures are indeed extreme or nearly extreme (by proving bounds) We organized two corresponding workshops during our stay at the Hausdorff Institute, with leading researchers working in the field.

- Workshop: Linear and semidefinite programming bounds (February 25 - 29, 2008)

Participants: Christine Bachoc, Eiichi Bannai, Henry Cohn, Jean Creignou, Herve Diet, Mathieu Dutour Sikirić, Dion Gijswijt, Gregory Kabatiansky, Abhinav Kumar, Oleg Musin, Fernando Mrio de Oliveira Filho, Pablo Parillo, Florian Pfender, Alex Samorodnitsky, Achill Schürmann, Makoto Tagami, Frank Vallentin

- Workshop: Experimentation with, construction of, and enumeration of optimal geometric structures (March 25 - 28, 2008)

Participants: David Bremner, Henry Cohn, Renaud Coulangéon, Michel Deza, Mathieu Dutour Sikirić, Viatcheslav Grishukhin, Jonathan Hanke, Abhinav Kumar, Jacques Martinet, Bertrand Meyer, Gabriele Nebe, Konstantin Rybnikov, Rudolf Scharlau, Achill Schürmann, Frank Vallentin, Stephanie Vance, Boris Venkov

During the three month program, we had the opportunity to work with two additional long term guests, David Bremner and Konstantin Rybnikov. The interaction with other groups of the Junior Semester Program was quite interesting and helpful. Moreover, the location of the Hausdorff institute allowed us to have several fruitful interactions with researchers from the University and the Max-Planck institute. For all of us, the three productive month at the Hausdorff institute were a great experience, for which we are very thankful!

Publications that arose from interactions at HIM

1. Henry Cohn, Noam D. Elkies, Abhinav Kumar, Achill Schürmann, Point Configurations that are asymmetric yet balanced, *Proceedings of the American Mathematical Society*, 138 (2010), 2863-2872.
2. Mathieu Dutour Sikirić, Olaf Delgado-Friedrichs, Michel Deza, Space fullerenes: computer search for new Frank-Kasper structures, *Acta crystallographica A*, 66 (2010), 602-615.
3. Mathieu Dutour Sikirić, Graham Ellis, Wythoff polytopes and low-dimensional homology of Mathieu Groups, *Journal of Algebra* 322 (2009), 4143-4150.
4. Mathieu Dutour Sikirić, Anna Felikson, Pavel Tumarkin, Automorphism group of root systems matroids, *European Journal of Combinatorics* 32 (2011), 383-389.

5. Mathieu Dutour Sikirić, Konstantin Rybnikov, Perfect but not generating Delaunay polytopes, accepted in special issue of *Symmetry Culture and Science on polytopes*, preprint at <http://arxiv.org/abs/0905.4555>
6. Mathieu Dutour Sikirić, Konstantin Rybnikov, Delaunay polytopes derived from the Leech lattice, preprint at <http://arxiv.org/abs/0907.0776>
7. Mathieu Dutour Sikirić, Achill Schürmann, Frank Vallentin, Complexity and Algorithms for Computing Voroni Cells of Lattices, *Math. Comp.* 267 (2009), 1713-1731.
8. Mathieu Dutour Sikirić, Achill Schürmann, Frank Vallentin, The contact polytope of the Leech lattice, *Discr. Comp. Geom.* 44 (2010), 904-911.
9. Mathieu Dutour Sikirić, Achill Schürmann, Frank Vallentin, Inhomogeneous extreme forms, preprint at <http://arxiv.org/abs/1008.4751>
10. Mathieu Dutour Sikirić, Yoshiaki Itoh, *Random sequential packing of cubes*, World Scientific, 2011.
11. Fernando Mário de Oliveira Filho, Frank Vallentin, Fourier analysis, linear programming, and densities of distance avoiding sets in R^n , *J. Eur. Math. Soc.* 12 (2010), 1417-1428.
12. Achill Schürmann, Perfect, Strongly Eutactic Lattices are Periodic Extreme, *Adv. Math.* 225 (2010), 2546-2564
13. Achill Schürmann, Enumerating perfect forms, in *Quadratic Forms – Algebra, Arithmetic, and Geometry*, AMS Contemporary Mathematics 437 (2009), 359-378.