REPORT ON THE GROUP "SINGULARITIES IN MAGNETIC AND SUPERCONDUCTING MATERIALS" WITHIN THE HIM JUNIOR TRIMESTER PROGRAM "ANALYSIS", SEPTEMBER-DECEMBER 2008

Participants. R. Ignat (Orsay), M. Kurzke (Bonn), C. Melcher (Oxford), D. Spirn (Minnesota)

Guests. R. Moser (Bath, two weeks); E. Miot (Paris 6, one week)

Scientific report. The quantum mechanical theories for some effects in solid state physics can be effectively described by continuum variational principles. We discussed theories that describe superconductors or ferromagnets, mostly focusing on singularities such as walls or vortices and their dynamical behavior. Specifically, we focused on vortices, point defects in two-dimensional vector fields that carry a topological degree. As topological features, these vortices are quite stable, and it is interesting to see how the natural dynamical laws for various physical situations generate motion of vortices.

Our first main result was a motion law for a toy model, a "mixed flow" for Ginzburg-Landau vortices combining properties of a nonlinear Schrödinger equation and a gradient flow [1]. We did the main work on this during an extremely fruitful visit of R. Moser to the HIM that was the basis for a lot of further extensions. Our result states that the vortex motion can be described by an ODE as long as the initial configuration satisfies a certain well-preparedness condition. Independently from our work, E. Miot had derived the same ODE in a very similar setting, and it was very nice that it was extremely easy to invite her to HIM so we could discuss what happens when that well-preparedness condition is relaxed.

Combining the ideas used to derive the motion law discussed in [1] with other arguments from geometric analysis, we were able to treat a more difficult system the Landau-Lifshitz-Gilbert equation, a geometric PDE that lies between the Schrödinger map problem and the harmonic map heat flow, and where the main problem is to control "bubbling" of harmonic spheres. In the pleasant atmosphere of HIM, we discussed many approaches to the bubbling problem until we were able to rule it out under some special assumptions, and obtained a motion law [2]. This research is still ongoing, and we have improved results now, all based on work done at HIM.

For a more realistic model of superconductors, we found a motion law by coupling our previous results with some PDE estimates on the magnetic field [3]. The main parts of that article were finished during our time at HIM.

We were also able to continue and initiate further work on rigorous understanding of static properties of ferromagnets, working with other scientists in Bonn [5] and elsewhere [4]. Some of this work is still in progress: in [6] we attempt to rigorously treat a situation occurring in thin magnetic films using variational convergence techniques. Our estimates extend and combine much of previous work in this area.

At HIM, we benefitted very much from an open and relaxed working atmosphere. We also enjoyed the interaction with the simultaneously present image processing group, who ran a far more ambitious guest program. Many of the researchers in that group were interested in variational methods, leading to quite helpful discussions. Among other things, the collaboration [7] has its roots in the meeting at HIM.

References

[1] Kurzke, M., Melcher, C., Moser, R., Spirn, D.: Dynamics for Ginzburg-Landau vortices under a mixed flow. *Indiana Univ. Math. J.* **58** (2009), 2597–2622.
[2] Kurzke, M., Melcher, C., Moser, R., Spirn, D.: Ginzburg-Landau vortices driven by the Landau-Lifshitz-Gilbert equation. *Arch. Rat. Mech. Anal.* **199** (2011), 843–888.
[3] Kurzke, M., Spirn, D.: Γ-stability and vortex motion in type II superconductors. *Comm. Partial Differential Equations* **36** (2011), 256–292.
[4] Ignat, R., Merlet B.: Lower bound for the energy of Bloch walls in micromagnetics, *Arch. Ration. Mech. Anal.* **199** (2011), 369–406.
[5] Ignat, R., Otto, F.: A compactness result for Landau state in thin-film micromagnetics, *Ann. Inst. H. Poincaré, Anal. Non linéaire* **28** (2011), 247–282.
[6] Ignat, R., Kurzke, M.: An effective model for boundary vortices in thin-film micromagnetics. In preparation.
[7] Kurzke, M., Mugnai, L.: The action functional for boundary transitions. In preparation.