Research group final report
for the HIM Junior Trimester Program on Stochastics Fall 2010

Disordered systems and extreme value theory

July 29, 2011

The research group aimed at collaborative research on disordered systems and extreme values of random fields on discrete and continuous structures. The group was composed of Louis-Pierre Arguin, Onur Gün, Zakhar Kabluchko, Nicola Kistler, and Anton Klimovskyi. The specific research objectives were first to describe the statistics of extreme values of particular stochastic processes that are important in statistical physics, the so-called *disordered systems*, and in probability to a broader extent. Such models include for example spin glass models, branching Brownian motion, and Gaussian processes with logarithmic correlations. These random fields can be seen as a collection of highly-correlated random variables, the *energies*, indexed by the *states* of the system. A second objective was to understand the dynamics of these systems. This latter problem can be translated in terms of random walks in random environment. (Here the states of the walk are the states of the system and the random environment are the random energies of the states.)

The junior research program at HIM has been a key moment for progresses in the objectives stated above. A list of works that are the direct consequences of the discussions and work performed at the HIM are listed below. We summarize here the contributions. Anton Klimovskyi's research during his stay was mainly concentrated around the analysis of the extremes of high-dimensional Gaussian fields with isotropic increments, that is whose correlations present a rotational symmetry. He proved and analyzed a computable saddle-point variational representation for the free energy in the infinite-dimensional limit [6, 7]. In particular, he gave a rigorous proof of an important formula due to Fyodorov and Sommers for the free energy of a particle in a rotationally invariant box. Zakhar Kabluchko has analyzed the distribution of the maximum of a family of Gaussian fields [4], those with $1/f^{\alpha}$-noise, that appear in many applications. He also investigated the statistics of values close to the maximum of processes with hierarchical correlations such as branching Brownian motion [5]. Louis-Pierre Arguin and Nicola Kistler worked mainly on branching Brownian motion (BBM), a Gaussian process with a tree-like correlation structure. Correlations of BBM are high, of the order of the variance, and it was unknown until then how the correlations affect the distribution of values close to the maximum. The work done at HIM unveiled, somewhat surprisingly, a Poissonian statistics close to the maximum (Poissonian statistics are typical of independent variables not correlated ones) [1]. The work has later led to a complete description of the statistics in terms of Poisson cluster process [2]. Finally, Onur Gün has made some progress with Véronique Gayrard on the dynamics of GREM models, a particular kind of spin glasses that is expected to present a good approximation of more physical models such as the Sherrington-Kirkpatrick models.

The aforementioned results could not have been possible without the support of HIM and its staff. They are the consequences of the interactions during the activities organized at HIM. An important one was the workshop *Disordered systems and extreme value statistics* we organized in September 2010. Twelve researchers from all over the world participated. Prof. Yan Fyodorov from the University of Nottingham gave a mini-course on Gaussian processes with logarithmic correlations that has been very influential for the discussions conducted in the rest of the semester. Another important activity was a mini-course given by Onur Gün and Nicola Kistler on *the statistics and dynamics of disordered systems*. Finally, all the participants of our group, and one guest Prof. Véronique Gayrard, have presented at the weekly seminars of the HIM junior program.

References

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[4] Kabluchko, Zakhar. The maximum of the Gaussian $1/f^{?}$-noise in the case $? < 1$. *Preprint 2010, arxiv.org:1010.1902*.

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