Report for HIM junior program on Analysis: Calculus of Variation and Image processing

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Starting from the well-known Mumford and Shah image segmentation model [3], and Total Variation minimizing (TV) functional [4],

$$\min \left\{ \int_\Omega |\nabla u| \, dx \;\; + \;\; \frac{\lambda}{2} \int_\Omega (u - u_o)^2 \, dx \right\} \tag{1}$$

variational model has been a crucial part of mathematical approaches to image processing. For instance, the Mumford-Shah [3] segmentation model,

$$\min \left\{ \alpha \mathcal{H}^1(\Gamma) + \beta \int_{\Omega \setminus \Gamma} |\nabla u|^2 dx + \int_\Omega (u_o - K * u)^2 dx \right\} \tag{2}$$

is a perfect example showing the importance of the variational models in imaging research as well as the significance in the calculus of variation research.

For the Analysis program, we explored various analysis aspects of imaging problems, by bringing together international researches who are actively doing research on imaging problems and/or analysis and calculus of variations. The following main six people organized the program on "Calculus of Variation and Image Processing". The four members were analysts: Marco Barchiesi (Carnegie Mellon University, USA), Massimiliano Morini (Sissa, Italy), Luca Mugnai (Max Planck, Germany), Marcello Ponsiglione (Roma, Italy), and two members focused on mathematical image processing: Sung Ha Kang (Georgia Institute of Technology, USA) and Triet Le (Yale University, USA). Together we have explored various analytical questions on different imaging problems.

Through this program, we have invited various internationally renewed researchers and promoted interactions. The HIM program was very helpful and positive about these collaborations which tremendously helped the success of the program. We have organized the weekly seminar open to any researchers from the nearby universities. The list of visitors includes Francesco Maggi (University of Firenze, Italy), Ivano Primi (University of Heidelberg, Germany), Matthias Roegar (Max Planck, Germany), Giuseppe Riey (Italy), Riccardo March (Rome, Italy), Ha Quang Minh (Germany), Haomin Zhou (Georgia Tech, USA), Giovanni Bellettini (Italy), Antonin Chambolle

(France) and Simon Masnou (U of Pierre, France). The location of the HIM is also ideal for various collaborations and we had various interactions with local researchers from University of Bonn and different research groups.

Couple of research article resulted through this program [2] and [1]: **Colorization** refers to recovering color of gray scale images when only small regions with color are given. The term "colorization" was introduced by Wilson Markle who first processed the gray scale moon image from the Apollo mission. Through this collaboration, we proposed using Reproducing Kernel Hilbert space (RKHS) for image and video colorization. The setting of RKHS and its extensions are widely considered in machine learning, and this is the first work applying RKHS framework to image and video colorization, and the vectorial settings of RKHS are analyzed in [2]. The explicit solution is superior compared to any iterative methods, and the flexibility of choosing different kernels allows easy colorization on texture image as well as cartoon images. Colorization results show big improvements in speed and quality. With a close connection to applied analysis, in [1] we proposed a new model of image segmentation which captures the oscillatory boundary while denoising the image. A typical segmentation model uses the total length penalization as $\mathcal{H}^1(\Gamma)$ for regularization, and this term is replaced by the area of the boundary neighborhood in the new model. This is an initiating work on capturing the fine details of the boundary in image segmentation problem. By replacing the length term with an area of the ϵ-neighborhood, this model allows the boundary with infinite perimeter, as long as it can be included in the ϵ bound. By computing the energy, it is shown that the model is able to denoise the image. Numerical experiments successfully capture the details of the oscillatory boundary while denoising homogeneous regions.

The HIM institute was truly ideal for mathematical collaborations and we were impressed with the support for research. The building: the office, many blackboards and many discussion/seminar rooms were quite convenient for mathematical discussions. The support for the visitors were essential for the success of the program, and all the staff member were very helpful. It was an impressive experience and we were all very grateful for all the HIM's support for research.

References

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