

Report on Research in Groups

Multiplicity problems in harmonic analysis

July 11 - August 31, 2012 and August 1 - 29, 2014

Organizers: Avraham Aizenbud, Joseph Bernstein, Eitan Sayag

Topics

The topics studied in our Program were motivated by the study of multiplicities in harmonic analysis as related to the Langlands program.

We interpreted multiplicities in the broad sense and considered qualitative aspects (finiteness) as well as quantitative aspects (study of matrix coefficients). Furthermore, we considered both local aspects (real and p -adic groups) as well as global aspects (multiplicities in the theory of automorphic forms). Finally, we considered homological aspects.

Goals

The goal of the research group was to obtain foundational results to the subject of multiplicities, such as finiteness results that would be general as well as studying specific aspects of the relations between multiplicities and the Langlands functoriality.

More specifically, some participants focused on studying the relation of multiplicities with theta correspondence and some with the Langlands-Arthur-Clozel base change. Others, studied general finiteness issues in the global setting.

Organization

The group met during August 2012 and August 2014. It consisted of our collaborators, students and postdocs. We worked mainly in pairs and had a weekly seminar where we discussed possible new directions of study.

Results

During the activity we were able to advance our understanding of multiplicities in Harmonic analysis in the following directions:

1. Aizenbud and Sayag, based on Bernstein's theory of the center of the category of smooth representations of p -adic groups, introduced a sheaf that control the multiplicities of families of representations in smooth representation. In particular they use this sheaf in [ASc] to study homological aspects of multiplicities.
2. Bernstein and Sayag formulated and proved a Harish-Chandra finiteness theorem regarding embedding to the space of automorphic functions for a families of (g, K) -modules.
3. Aizenbud and Sayag initiated the study of distributions on spherical spaces that are finite with respect to the Bernstein center. These studies led to a new method to approach invariant distributions and is expected to have more applications in the theory of Gelfand pairs.
4. J. Bernstein initiated and developed his theory of Stacks in representation theory.
5. J. Bernstein worked on his project with A. Reznikov on periods of automorphic forms.
6. A. Mitra and C. Venkatasubramanian, following a suggestion of Sayag, studied the relation of base change in the Langlands program and multiplicity spaces for the pair (GL_n, GL_{n-1}) .

The program had the following outcomes:

- Final Draft of the text Cohen Macaulay Properties in representation theory (see [ASa]).
- Initiation of a project on freeness of certain Hecke modules of geometric origins (see [ASb]).
- Initiation of a project on homological multiplicities (see [ASc]) with a proof finiteness in the symmetric case.

- Final version of the work on counting representations of arithmetic groups (see [AAb]).
- Work on geometric realization of Jacquet modules (see [Yom]).
- Work on multiplicities in the space of Automorphic forms (see [BS]).
- Advance in a project on Categorification of Hopf algebras (see [GG]).
- Initialization of the study of z -finite distributions on p -adic groups (see [AGS]).
- Work on a project on multiplicity spaces, Global Automorphic periods and L -functions (see [BR]).
- Initiation of the study of representation of Stacks over p -adic fields (see [Ber]).
- Work on Reciprocity laws and K -theory (See [MY]).
- Work on Distinction of representations by Levi subgroups and theta lift (See [SV]).
- Jiajun Ma worked on theta lifts of cuspidal representations (with H. Loke, See [MaLoke], [MaLoke]).
- Work on Distinction and Bernstein center (See [MS]).
- Initiation of a project on base change and distinction (See [MV]).
- Work on a paper on decay of matrix coefficients (See [KSS]).

References

- [AAb] Aizenbud, A.; Avni, N. *Counting points of schemes over finite rings and counting representations of arithmetic lattices*, Online version at arXiv:1502.07004.
- [ASa] Aizenbud, A.; Sayag, E. *On a Cohen-Macaulay Property Of Spherical Varieties*, Online version at <http://www.wisdom.weizmann.ac.il/~aizenr/2Publications/CM.pdf>

- [ASb] Aizenbud, A.; Sayag, E. *A Short proof of Hironakas Theorem on freeness of some Hecke Modules*, Online version at <http://www.wisdom.weizmann.ac.il/~aizenr/2Publications/Hir.pdf>
- [ASc] Aizenbud, A.; Sayag, E. *Homological multiplicities in representation theory*, preprint.
- [AGS] Aizenbud, A.; Gourevitch, D.; Sayag, E. *z -Finite distributions on p -adic groups*, Advances in Mathematics, Volume 285, 5 November 2015, Pages 13761414.
- [Ber] Bernstein, J. *Stacks in Representation Theory. What is a continuous representation of an algebraic group ?*, Online version at arXiv:1410.0435.
- [BR] Bernstein, J.; Reznikov, A. *Periods and global invariants of automorphic representations*, Online version at arXiv:1312.2898.
- [BS] Bernstein, J.; Sayag, E. *Harish-Chandra finiteness theorem in families*, Draft.
- [GG] Gal, A.; Gal L. *Symmetric self-adjoint Hopf categories and a categorical Heisenberg double*, Online version at arXiv:1406.3973v3.
- [KSS] Krotz, B.; Sayag, E.; Schlichtkrull, H. *Decay of matrix coefficients on reductive homogeneous spaces of spherical type*, Math. Z. (2014) 278:229249, DOI 10.1007/s00209-014-1313-7.
- [MaLoke] Ma, J.; Loke, H. *Local Theta Correspondences of Supercuspidal Representations*, Online version at arXiv:1512.01797.
- [MaLoke] Ma, J.; Loke, H. *Local theta correspondences between epipelagic supercuspidal representations*, Math. Z. (2016) 283:169196, DOI 10.1007/s00209-015-1594-5
- [MV] Mitra, A.; Venkatasubramanian, C. *Base change and $(GL_n(F), GL_{n-1}(F))$ -distinction*, J. Ramanujan Math. Soc. 31, No.2 (2016) 109124
- [MS] Mitra, A.; Sayag, E. *Sp -Distinction and Bernstein Components*, 12 page Draft.

- [MY] Musicantov, E.; Yom-Din, A. *Reciprocity laws and K-theory*, Online version at arXiv:1410.5391v3.
- [SV] Sayag, E.; Venkatasubramanian, C. *Representations of $GL(n)$ distinguished by $GL(k) \times GL(n - k)$* , 20 page Draft.
- [Yom] Chen, T. H.; Yom-Din, Alexander, *A Formula for the Geometric Jacquet Functor and its Character Sheaf Analogue*, Online version at arXiv:1507.00606v1.