

Report on the Hausdorff Trimester Program

# Harmonic Analysis and Partial Differential Equations

May 5 - August 22, 2014

**Organizers:** Herbert Koch, Daniel Tataru, Christoph Thiele

## Topics

The topics of the program included partial differential equations with focus on behavior of solutions of dispersive equations, fluids, and elliptic problems. Topics also included real and harmonic analysis with focus on Fourier multipliers, singular integrals and other scale invariant problems, applications in particular to partial differential equations but also to ergodic theory and number theory.

## Goals

Harmonic analysis and partial differential equations have been closely inter-linked areas in recent decades, with ideas flowing back and forth and stimulating progress in both areas. The goal of the program was to bring together researchers in these two fields, some who have worked in both fields and some who have specialized in one field but are interested in learning more about the other. The aim is to strengthen existing links between the areas and open new lines of research, making use of the many possible ways in which new developments in one area impact the other.

## Organization

Throughout the trimester, the program provided a stimulating atmosphere to participants for collaborative research. Outside the workshops, approximately one lecture was scheduled almost every workday, followed by tea-time. These two were the main social focus on a daily basis, the remaining time was left for research and collaboration.

Approximately once a month a week long workshop was held:

- The Introductory Workshop "Topics in Harmonic Analysis and PDEs", May 12-16, 2014, was attended by 29 participants. Lecture series were given by Sebastian Herr, Alexandru Ionsecu, Camil Muscalu, and Alexander Volberg.
- During the week of May 25-30, 2014, a Spring/Summer School "Carleson theorems and Radon type behavior" with 17 participants was organized by Lillian Pierce, Po-Lam Yung, and Christoph Thiele. Every Participant gave two 50min lectures. The participants were graduate students or postdocs and not necessarily an expert on the topic, but interested in jointly learning the subject.
- The first Workshop "Harmonic Analysis Methods in Dispersive PDEs", June 10-13, 2014, with 49 participants was organized by Herbert Koch and Daniel Tataru.
- The second Workshop on Real Analysis July 14-18 with 73 participants was organized by Herbert Koch and Christoph Thiele. This workshop was particularly heavily attended, since it happened simultaneously with a closely related conference on number theory, and had a joint session with that conference.
- The Closing Workshop, August 4-8, 2014 with 45 participants was organized by Herbert Koch, Daniel Tataru, Christoph Thiele.

Moreover several events were organized in the vicinity of the HIM program and attended by many participants of the program, for example Daniel Tataru gave a series of six Lipschitz lectures and Herbert Koch and Stefan Müller organized a Mini-workshop on Euler equation and turbulence.

The program had a total of 144 participants, with attendance peaking at the workshops.

## Results

The homepage of the trimester listed the recorded talks, the publications, and preprints. Forty preprints are listed there, but it is known that more results were generated from the program.

Of the many contributions it is impossible to go into detail on all, we focus on a selection where the collaborator team had not published before the HIM

program and can be considered unlikely to have come together without the program.

1) Andreas Seeger and Tino Ullrich started a collaboration on a result on Haar projection numbers and failure of unconditional convergence in Sobolev spaces, which later turned into a second preprint on deterministic examples for lower bounds for Haar projections.

2) Rowan Killip, Jason Murphy, and Monica Visan, discovered solutions addressing the final-state problem for the cubic-quintic NLS with non-vanishing boundary conditions

3) Stefan Buschenhenke, Detlef Müller, and Ana Vargas proved a Fourier restriction theorem for a two dimensional surface of finite type.

4) Ioan Bejenaru, Zihua Guo, Sebastian Herr, and Kenji Nakanishi proved well posedness and scattering for the Zakharov system in four dimensions.

5) A young collaborator team formed with Kevin Hughes, Ben Krause, Mariusz Mirek, Bartosz Trojan, and Pavel Zorin-Kranich in various configurations, which resulted in a sequence of papers related to convergence questions on maximal operators and ergodic means.

## **Acknowledgements**

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