Report on the Hausdorff Trimester Program

Combinatorial Optimization

September 1 – December 18, 2015

Organizers

András Frank (Budapest)
Satoru Iwata (Tokyo)
Jochen Könemann (Waterloo)
Jens Vygen (Bonn)

Topics

Combinatorial Optimization (CO) [14, 20, 27] is an immensely active field with many practical applications. Its focus is on the design of algorithms for finding specified configurations or objects in finite structures such as directed and undirected graphs, hypergraphs, networks, matroids, partially ordered sets, and so forth. The goal is often to find an optimum (or at least near-optimum) such configuration with respect to a specified objective function. The design of such algorithms can rarely be done in an adhoc, simple-minded fashion, but rather requires the unravelling of the problem's inherent combinatorial structure.

This trimester program focused on four key areas: (i) connectivity, routing, and network design; (ii) rigidity, submodularity, and discrete convexity; (iii) relaxations and polyhedral methods; and (iv) algorithmic and computational game theory. These areas are connected to each other in various ways.

Goals

The overall goal in the design of the trimester program was to provide a stimulating collaborative environment for leading experts and younger researchers in the field and establish new and deepen existing international collaborations. This should generate momentum which would ultimately lead to advances for some of the salient open questions in CO.

Organization

The organizers carefully chose the four focus areas mentioned above and the set of invitees. For each of these focus areas, we crystallized a collection of prominent open questions and key experts that we invited to Bonn.

There were five special weeks during the trimester program, a summer school preparing especially junior participants, and four workshops, each focusing on one of the focus areas, and each also with leading experts as additional co-organizers:

- **Summer school**. Speakers: Eden Chlamtac (Ben Gurion, Lift & Project and Integrality Gaps), James Lee (University of Washington, Semidefinite Extended Formulations and Sums of Squares), Kazuo Murota (Tokyo, Discrete Convex Analysis), Heiko Röglin (Bonn, Smoothed Analysis), Zoltán Szigeti (Grenoble, Connectivity Problems).

- **Connectivity, Routing, Network Design**. Organizers: András Frank (Budapest), Bruce Shepherd (McGill).

- **Rigidity, Submodularity, Discrete Convexity**. Organizers: Satoru Iwata (Tokyo), Tibor Jordán (Budapest) , Jan Vondrák (IBM).

- **Relaxations and Polyhedral Methods**. Organizers: Nikhil Bansal (Eindhoven), Ola Svensson (EPFL), Jens Vygen (Bonn), David Williamson (Cornell).

- **Algorithmic and Computational Game Theory**. Organizers: Jochen Könemann (Waterloo), Ruta Mehta (UIUC), Heiko Röglin (Bonn), Bernhard von Stengel (LSE)

Outside these five special weeks, we organized a regular seminar with approximately two talks per week. (Partial) results obtained during the trimester program were presented immediately. There were also several open problem sessions. Talks during the workshops and also outside were very well attended, sometimes approaching the capacity of the seminar room. Most of them were recorded. There were also informal meetings every day, and the atmosphere in the institute was extremely conducive to foster joint research.

Results

Many new collaborations have been established during the trimester program. These groups, which have never worked together before, discovered new connections and obtained interesting new results on several fundamental problems in CO. We describe a few highlights below; see the website for a more complete list of publications that resulted from this trimester program.

Traveling salesman problem

The traveling salesman problem (TSP) is probably the most famous combinatorial optimization problem. It is notoriously hard. One of the most prominent problems in CO is to improve on Christofides' 1976 algorithm with approximation ratio $\frac{3}{2}$. The leading TSP experts were present for a long period and working together, including Bill Cook, Gérard Cornuéjols, Michel Goemans, and many others.

For the s-t-path TSP (where the endpoints of the tour are not identical), we know how to improve on Christofides, but have not attained yet the conjectured lower bound. However, almost all experts on this problem were in Bonn during the trimester program, and significant progress was made by Corinna Gottschalk, András Sebő, Jens Vygen, and Anke van Zuylen. The techniques developed during the trimester, including designing a convex combination with strong extra properties [16] and deleting edges before parity correction [28], will surely have more impact in the future.

Also due to these successes obtained in Bonn, BIRS (Banff) will devote a whole workshop on the TSP in late 2018.

Moreover, new ideas for better computational work resulted from the program, mainly by Bill Cook and Stefan Hougardy; this is still work in progress. Moreover, a new variant called "k-trails" was suggested by András Sebő in the beginning of the trimester program and resolved subsequently by Mohit Singh and Rico Zenklusen [29]. Finally, significant progress on a related ordering problem known as Lovász' gasoline puzzle was made by Alantha Newman, Heiko Röglin and Johanna Seif [24].

Network design

Ensuring and maintaining connectivity is vital in most practical networks. Not surprisingly therefore this is a central topic that still attracts much attention in CO. There are many long-standing unresolved open questions of which we tackled some.

So far, the best approximation algorithms for designing networks with sufficient connectivity were using the ellipsoid method and iterative rounding; no combinatorial algorithms were known, until Andreas Feldmann, Jochen Könemann, Kanstanstin Pashkovich, and Laura Sanità [13] found such an algorithm during the trimester program. Interestingly, they use the multiplicative weight update method, which has been used also in completely different contexts during the trimester.

The most prominent special cases of the general network design problem are Steiner trees and Steiner forest. They are often considered in a prize-collecting generalization, where one pays a penalty for not connecting a terminal pair. The natural linear programming relaxation for prize-collecting Steiner forest was conjectured to have an integrality gap of 2. Könemann, Olver, Pashkovich, Ravi, Swamy, and Vygen [18] started working together on these problems during the trimester and disproved this conjecture, providing an intricate family of instances with gap at least $\frac{9}{4}$. They also showed that the popular belief in the existence of an iterative rounding based algorithm for prize-collecting Steiner tree via the natural LP is false. Similar progress was made by Friggstad, Könemann and Shadravan [15] on the directed Steiner tree problem.

A dual question relates to minimum cuts. While they were well understood in graphs, Chandra Chekuri and Chao Xu [10] extended our knowledge to hypergraphs, found better algorithms and revealed a hypercactus structure. Matthias Mnich, Virgina Williams and László Végh [22] attacked the directed feedback vertex set problem, where one wants to remove a minimum-weight set of vertices from a directed graph so that the resulting graph is acyclic. The outstanding open question in this area is to find a constant-factor approximation. The authors address the interesting special case where the graph is a tournament and give a $\frac{7}{3}$-approximation algorithm, breaking the long-standing threshold of $\frac{5}{2}$.

Neil Olver, who worked on the important but very difficult generalized virtual private network design problem before, raised a lot of interest during the trimester program, creating new collaborations and progress with Kristóf Berczi, Michel Goemans, Jannik Matuschke, András Sebő, Jens Vygen, and others. There are only partial results so far, but the collaboration will continue.

Last but not least, Olver and Végh [25] obtained the (probably) ultimate solution for the important generalized flow problem. Their seminal work started during the trimester program.

Rigidity, Submodularity, and Discrete Convexity

Motivated by questions from physics and material sciences, as well as by mathematical applications in areas such as sphere packings, the rigidity analysis of infinite periodic frameworks has seen an increased interest in recent years. A challenging open problem in this context is to extend Jackson-Jordán's characterization for the global rigidity of finite graphs to that of infinite periodic graphs. At HIM, Bernd Schulze and Shin-ichi Tanigawa worked on this problem, and gave an answer in a certain rigidity model [17]. The proof crucially exploits the submodularity of counting functions on the underlying group-labeled graphs.

Bill Jackson, Anthony Nixon, Bernd Schulze, and Shin-ichi Tanigawa continued a joint work that had started at the Banff rigidity workshop 2015 with Yaser Eftekhari and Walter Whiteley. During the stay at HIM, they found a new rigidity transfer technique among various rigidity models [12]. As a corollary they found an extension of Jackson-Jordán's theorem on rigidity with linearity constraint as well as variants of Jackson-Owen's theorem about point-line rigidity.

During their stay in Bonn, Kristóf Bérczi and András Frank [7, 5, 6] made essential steps toward obtaining new types of (exact) min-max results and good characterizations. An important novelty of their approach is that they obtained these results in an environment where the natural weighted (or min-cost) extensions of the corresponding problems are already NP-hard. For example, they solved the packing problem of branchings with specified sizes, significantly extended a classic result of Ryser on maximum term rank, and characterized the degree-sequences of simple k-node-connected directed graphs.

Just before the trimester program, Lee, Sidford, and Wong obtained a surprising breakthrough result on submodular function minimization [21]. They provided substantial improvements with the aid of the fast cutting plane method. The resulting bound on the number of function evaluations is $O(n^3 \log n)$, where n is the cardinality of the ground set. This work was first presented in Bonn by Yin Tat Lee and Aaron Sidford and triggered a lot of discussions. During their stay in Bonn, they obtained further progress together with Deepanarb Chakrabarty, resulting in a subquadratic number of oracle calls [8]. Another work that was started in Bonn was the collaboration of Kazuo Murota and Fabio Tardella on scaling algorithms for integrally convex functions [23].

Cuts, clutters, and extended formulations

Ideal and Mengerian matrices sit at the heart of the field of packing and covering. Testing the two properties is co-NP-complete. However, during the trimester program, Ahmad Abdi, Gérard Cornuéjols and Kanstanstin Pashkovich showed that detecting *degenerate projective plane* minors in A can be done in polynomial time. This result is very surprising because degenerate projective planes are most often the reason why A is not ideal, by a seminal result of Lehman. Using the polynomial time detection algorithm, the group gave quite compelling evidence in support of a 20 year old question of Conforti and Cornuéjols whether non-Mengerian matrices A can be certified by $0, 1, +\infty$ vectors. In particular, they gave a characterization of ideal minimally non-packing clutters, similar to that of Lehman for minimally non-ideal clutters, and showed that those with covering number 2 have a central role. These results are a summary of the two papers [1, 2] written at HIM.

The knapsack problem is, in a sense, the easiest NP-hard problem; its study has been fundamental for general integer programming. Bazzi, Fiorini, Huang and Svensson [4] settled a long-standing open question in this area by developing a quasi-polynomial size extended formulation whose gap asymptotically equals that of the knapsack-cover inequality strengthened natural formulation. The new formulation uses a novel connection between monotone circuits and extended formulations.

There was also a lot of activity on cut-generating functions, but this is still wortk in progress. Last but not least, Daniel Dadush, László Végh, and Giacomo Zambelli [11] developed new algorithms for linear programming in Bonn.

Algorithmic and computational game theory

Karthekeyan Chandrasekaran and Jochen Könemann worked with Britta Peis and three of her students on graph stabilization questions. In their work [9] they focus on a fine-grained stabilization strategy, namely stabilization of graphs by fractionally increasing edge weights. As a main result, the group shows that their stabilization problem is (nearly) as hard to approximate as the notorious densest k-subgraph and set-cover problems. These are the first super-constant hardness of approximation results for stabilization problem.

Könemann, Pashkovich and Tóth [19] studied the classical stable matching problem and found a novel 2-page alternative proof of Rothblum's famous integrality result for the stable matching polytope. Ágnes Cseh, Jannik Matuschke and collaborators studied a natural and practically important variant of the classical matching problem with lower and upper bounds; the authors provide a comprehensive study of the approximability and complexity landscape surrounding this problem in [3].

Georgios Piliouras and two collaborators studied the application of the ubiquitous multiplicative weights update method with arbitrary admissible constants as learning rates to problems in the class of congestion games. In [26], the authors establish novel connections between multiplicative weights update and the Baum-Welch algorithm to prove convergence to exact Nash equilibria.

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