Report on the project 'Intrinsic meaning of local gauge invariance' at the Hausdorff Research Institute for Mathematics, Bonn

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1 Research subjects, new results, future directions

The group consisted of four researchers: Katarzyna Rejzner, Jan Schlemmer, Yoh Tanimoto and Wojciech Dybalski. During our stay at the HIM we discussed and worked on the following questions related to the problem of local gauge invariance (among others):

1. *Classical Electrodynamics and directional charges.* At the heuristic level, directional charges arise from local gauge symmetry via the Noether theorem. (Similarly to the electric charge, which arises from global gauge symmetry). We established their existence rigorously in a system describing a charged particle interacting with the electromagnetic field. Moreover, it turned out that the directional charges are a promising tool for asymptotic analysis of solutions of the Maxwell equations – a fact which apparently escaped attention so far. Heuristically, their conservation enforces slow, Coulomb-like decay of the radiation emitted by an accelerating charge. This observation should allow further development of scattering theory of Maxwell equations coupled to charged particles presented in [9].

2. *Quantum Electrodynamics and infraparticle problem.* It is well-known that a charged particle does not have a sharp mass in generic representations of Quantum Electrodynamics as it is always accompanied by soft photons. It has, however, been recently conjectured by Buchholz and Roberts that there exist special charged representations where this problem disappears. We have constructed such representations in the framework of non-relativistic Quantum Electrodynamics. It turned out that they have rather exotic features: Neither the position operator of the charged particle nor the asymptotic photon fields exist, which invalidates the conventional scattering theory.

3. *Non-relativistic Quantum Electrodynamics and directional charges.* It is well known that quantization of gauge theories requires an (arbitrary) choice of gauge to eliminate superfluous degrees of freedom. The question if different choices lead to the same quantum theory appears to be unanswered to date. Our conjecture is that this is not the case: Different quantization prescriptions may give rise to different values of directional charges. A convenient framework to verify this conjecture is that of non-relativistic Quantum Electrodynamics. We expect that its

quantization in the Coulomb gauge and in an axial gauge will give rise to different values of directional charges and therefore to different quantum-mechanical theories. While quantization in the Coulomb gauge is well known [9], quantization of non-relativistic Quantum Electrodynamics in axial gauges remains to be performed, e.g. following [7].

Apart from the problem of local gauge invariance we worked on several other aspects of quantum field theory. Papers completed during our stay at the HIM concern the problem of asymptotic completeness [3, 2] and construction of quantum field theories [6, 10].

2 Organization of HIM activities

1. *Workshop 'Algebraic Quantum Field Theory and Local Symmetries'*, 26.09.2012–28.09.2012. The workshop gave an overview of recent developments toward mathematical understanding of the problem of local gauge invariance. The speakers include K.Fredenhagen and R.Longo.

2. *AQFT seminar*. From October 2012 we organized a series of talks almost on a weekly basis on AQFT and related fields. The speakers include three of us (W.D., J.S., Y.T.) and A.Pizzo. Several members of other groups and some researchers in Bonn attended our seminars.

3. *Trimester Seminar*. Jointly with other groups we orgnized a weekly seminar. Three of us (K.R., J.S., Y.T.) and J.Zahn gave talks.

3 Later work

After the end of the program the group members published several papers with an acknowledgement of their stay at the HIM. These works concern the infrared problem in non-relativistic QED [4, 5], local gauge invariance in perturbative relativistic QFT [8], construction of QFT models [11, 1] and the problem of non-interaction in conformal field theory [12].

4 Conclusion

It is clear from the preceding sections that the HIM offers an excellent environment for scientific activity. We enjoyed our stay at HIM and we are looking forward to future events at the HIM in the field of Mathematical Physics.

References

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