Final report for the Junior Hausdorff Trimester Program "Symplectic Geometry and Representation Theory", 02.Oct.2017–21.Dec.2017

Key information

Group: Braids, webs, affine Grassmannian, and 'higher' representation theory.

Research areas: Geometry, representation theory and topology, more specifically geometry of Grassmanians, Lagrangian submanifolds, braid and knot theory, link homology, TQFT, tensor categories, categorification, 2-representation theory, modular representation theory.

Members: Martina Balagovic, Michael Ehrig, Agnès Gadbled, Jonathan Grant, Peter McNamara, Peng Shan, Anne-Laure Thiel, Daniel Tubbenhauer, Emmanuel Wagner, Paul Wedrich, Arik Wilbert, Oded Yacobi.

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Our research group, consisting of the members named above, participated in the Junior Hausdorff Trimester Program "Symplectic Geometry and Representation Theory" in the fall 2017. We were based in the main building of the HIM which made interactions within our group very easy. We also enjoyed the discussions with the other two groups, the Institute providing a vibrant and exciting atmosphere. On the whole the synergy of the three groups was really amazing and was made possible by the choice of the scientific theme of the trimester made by the HIM. This theme was indeed on the edge of recent mathematical developments and allowed to bring together people from diverse scientific communities.

More specifically the main goal of our research group was to get a better understanding of the interplay between symplectic geometry and representation theory and their applications to link homologies and categorification. At the cornerstone of these two fields of mathematics lies Khovanov's famous link homology, which turns up, in one way or another, in a lot of aspects of modern geometric or algebraic representation theory.

In the inspiring atmosphere instilled by the HIM, this main aim of ours turned out to lead to very fruitful collaborations with various (quite diverse) ideas coming together. Our group and its guests have studied the following topics (among others): geometric Satake correspondence, symplectic duality, Coulomb branches of certain quiver gauge theories, quantum representation theoretical aspects as e.g. webs and link homologies, categorified quantum groups and their 2-representations, categorical representations of braid groups, super algebras and their representations, just to name a few.

We would like to thank the HIM for the opportunity to participate in this Junior Hausdorff Trimester Program. This unique chance to interact with mathematicians from many different research areas was much appreciated.

Winter school and workshop

We had two very intense weeks around the end of November with a winter school and a follow-up workshop, both entitled "Categorification, representation theory and symplectic geometry".

- The winter school had five main speakers, Eugene Gorsky, Yankı Lekili, Andrew Mathas, Volodymyr Mazorchuk, Catharina Stroppel, from all around the world. Most talks were recorded by the HIM and are available online. With about 80 participants, mostly students and postdocs, the winter school was a very

successful and lively activity. The participants were given the opportunity to present their research during a poster session. This was a great success with around 20 posters displayed. We also offered the authors of posters to give a short presentation, which most of them happily did.

- The follow-up workshop, with about 70 participants, enabled to strengthen and deepen the ideas broached during the winter school through several talks by international experts in the field such as Joseph Grant, Sira Gratz, Marco Mackaay, Vanessa Miemietz, James Pascaleff, Krzysztof Putyra, Gwyn Bellamy, Konstanze Rietsch, Louis-Hadrien Robert, Nicolò Sibilla and Joshua Sussan. Most talks were recorded by the HIM (and are available online) and we also had organised a live streaming throughout the course of the workshop which was screened in the secondary seminar room of the Institute.

Visitors and international interaction

During the program our group had plenty of visitors who came for a week or longer and contributed to the trimester program through collaborations, discussions, giving seminars and guest lecture series.

- Giving a small lecture series: Chris Elliott, Thomas Gobet, Owen Gwilliam, Pedro Vaz.
- Giving lectures during the winter school: Andrew Mathas, Volodymyr Mazorchuk.
- Others: Gwyn Bellamy, Daniele Faenzi, Marco Mackaay, Vanessa Miemietz, Anna Mkrtchyan, Louis-Hadrien Robert, Nicolò Sibilla, Wai-kit Yeung.

(This list does not include all people participating in the winter school or the workshop for only one week.)

Seminars

We ran four types of seminars during the trimester program, which we organized together with the other two groups.

- Small lecture series which consisted in talks by international experts; we had four of these with a total of 12 talks. For all the series, lecture notes or videos can be found on the HIM website of the program.
- The Trimester seminar which consisted in talks given by guests or participants. There were 8 seminars in total, all of which are again available online.
- An informal seminar, held every Monday, given by the participants of our and the other groups to strengthen interactions.
- Various reading seminars, held every Monday, e.g. on aspects of Coulomb and Higgs branches. The speakers were the participants of the various groups.

Follow-up events

There were several events organized in the spirit of our very successful winter school and workshop, which would not have taken place without our

- Nils Carqueville, Anton Mellit and **Paul Wedrich** organised a workshop in January 2019 that followed up on the topics of the program: http://www.categorification.net/esi19.
- *Daniele Faenzi*, **Agnès Gadbled**, **Anne-Laure Thiel** and **Emmanuel Wagner** discussed braid group operations on exceptional sequences via mutations. The ideas brewed then led us to organize a follow-up workshop in Dijon in September 2018: https://extresses.sciencesconf.org/.
- **Daniel Tubbenhauer**, motivated by the huge success of the program, organized a conference in September 2018 during which quite a few participants or guests of the Junior Trimester program participated: http://www.dtubbenhauer.com/conference2018.html.

Hard research outcome

The following is the list of the research outcome of members of our group: new collaborations, projects, follow-ups, work in progress, (pre)publications. All of the below was carried out or initiated during the program (papers, of course, take some time to be completed). This list does not include the various very useful informal discussions which we had during the program. Participants of our group are in bold, our guests in italic.

- **Martina Balagovic**, Iva Halacheva, Emily Norton and Catharina Stroppel discussed the affine VW supercategory, which is used in describing the representation theory of the periplectic Lie superalgebra $p(n)$, and proved a basis theorem for it. [B+18]
- Jean-François Barraud, **Agnès Gadbled**, Roman Golovko and Hông Vân Lê constructed a Novikov version of the fundamental group which is the first step of a project to develop a Novikov–Floer fundamental group in symplectic geometry. [BGGL17]
- **Michael Ehrig** and **Daniel Tubbenhauer** developed a generalization of cellular algebras which might turn out to be useful in representation theory in the near future. [ET17]
- **Michael Ehrig** and **Daniel Tubbenhauer** studied, in a follow-up to their paper on relative cellular algebras, certain properties of certain path algebras. [ET18]
- **Agnès Gadbled**, **Anne-Laure Thiel** and **Emmanuel Wagner** explored the symplectic aspects of a categorical action of the braid group of the cylinder in a paper very near completion.
- *Thomas Gobet* and **Anne-Laure Thiel** constructed and studied some generalizations of the category of Soergel bimodules, in the case of Coxeter group of type A_2 in [GT18a] and in the case of cyclic groups in [GT18b].
- *Eugene Gorsky* and **Paul Wedrich** continued their collaboration on categorical invariants of annular links in the context of Hilbert schemes that led to the preprint [GT19].
- Joel Kamnitzer, Peter Tingley, Ben Webster, Alex Weekes and **Oded Yacobi** studied categorified tensor products via a version of category $\mathcal{O}$ associated to affine Grassmannians. [KWWY18]
- *Marco Mackaay*, *Volodymyr Mazorchuk*, *Vanessa Miemietz* and **Daniel Tubbenhauer** used the quantum Satake equivalence to study 2-representations of Soergel bimodules. [MMMT18]
- *Marco Mackaay*, *Volodymyr Mazorchuk*, *Vanessa Miemietz*, **Daniel Tubbenhauer** and *Xiaoting Zhang* started a project to classify all 2-simples of Soergel bimodules, and the paper is expected to appear on arXiv in the Summer 2019.
- **Peter McNamara** proved the monoidality of Kato's reflection functors in the theory of KLR algebras. [McN1]
- **Peter McNamara** found the first examples of non-perverse parity sheaves on the Schubert variety for $p > 2$ in an infinite family of examples. [McN2]
- Loïc Poulain d'Andecy, **Anne-Laure Thiel** and **Emmanuel Wagner** studied the Birman–Murakami–Wenzl algebras in [PdATW17]. They made explicit new bases on this tower of algebras, which are constructed inductively in the same spirit as for the Hecke algebras. They also classified transverse Markov traces on the BMW algebras.
- Hoel Queffelec and **Paul Wedrich** authored two papers on extremal weight projectors [QW18a] and the categorification of skein algebras of surfaces using Khovanov homology [QW18b].
- *Louis-Hadrien Robert* and **Emmanuel Wagner** completed their joint project on symmetric link homologies which appeared just after the end of the semester as a preprint. [RW18]

- *Louis-Hadrien Robert* and **Emmanuel Wagner** started their project on a combinatorial and algebraic categorification of the Alexander polynomial which appeared one year later as the preprint. [RW19]
- Neil Saunders and **Arik Wilbert** studied the geometry and topology of exotic Springer fibers for orbits corresponding to pairs of one-row partitions. [SW18]
- Marko Stosic and **Paul Wedrich** released a preprint that proves the conjectured "Knots-Quivers" correspondence of Kucharski–Reineke–Stošić–Sułkowski for 2-bridge links. [SW17] This was presented and discussed at the programme's workshop and has led to an interpretation in terms of symplectic field theory that will lead to follow-up publications.

on behalf of our group,
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