Quantum geometric and algebraic representation theory

Final Report

Quantum Geometric and Algebraic Representation Theory was one of three constituent groups of the junior Trimester program on *Symplectic Geometry and Representation Theory* during fall 2017.

The group members were: Alexander Caviedes Castro, Peter Crooks, Sam Gunningham, Iva Halacheva, Benjamin Harris, David Jordan, Cris Negron, Maarten van Pruijssen, Travis Schedler, José Simental Rodríguez, Tobias Weich, Yaping Yang, Gufang Zhao. In addition, a number of graduate students were working directly with the group members during the trimester, incuding Daniel Kaplan, Anna Mrktchyan, Tim Weelink, and Michael Wong. The group leaders were Sam Gunningham and Travis Schedler.

The group brought together researchers from various different areas of representation theory (including analytic, algebraic, combinatorial, and geometric flavours) each with a common interest in symplectic and Poisson geometry.

1 Visitors

We invited various visitors over the course of the trimester program to discuss and collaborate with the program members. In some cases, the visitors gave talks in one of the seminar series.

The visitors associated with our group included: Gwyn Bellamy, Michel Van den Bergh, Yannick Bonthonneau, Julia Budde, Iordan Ganev, Wahei Hara, Yoshiki Oshima, Guido Pezzini, Brent Pym, Pavel Safronov, Changjian Su, Wai-Kit Yeung.

2 Workshop

The group organized a workshop from 16th to 18th October. There were 53 registered participants, including many graduate students and junior researchers who had travelled to Bonn to attend. Talks were given by: Gwyn Bellamy, Ben Davison, Tina Kanstrup, Allen Knutson, Martina Lanini, Penghui Li, Michael McBreen, Yoshiki Oshima, Olivier Schiffmann, Jan Schröer, Peng Shan, Wolfgang Soergel, Špela Špenko, Michel Van den Bergh, and Yaping Yang.

Many of the talks were recorded by the HIM and are available online. There was also a live stream of the talks displayed in another room, allowing for a greater number of participants.

3 Internal Seminar Series

Along with the Trimester seminar organized with the other groups, our group held a regular internal seminar series where members of the group gave 20 minute talks introducing their research topics to the other program participants. A number of the group members felt that these talks did a good job building links between the various research areas in the group and stimulating discussion.

4 Research Output

Below we give an overview of the progress made by the researchers in our group that was facilitated by the HIM trimester program.

- Peter Crooks and Iva Halacheva studied instances of resolutions of singularities in the context of closures of highest weight orbits (in progress). Since the trimester, they have also extended our collaboration to study a new framework for Hessenberg varieties and completely integrable systems.
- Peter Crooks and Maarten van Pruijssen used spherical geometry to solve the non-emptyness problem for a large class of hyperkhler quotients. Their work has resulted in the preprint [CvP19].
- Sam Gunningham and David Jordan continued their project on quantum D-modules. The background work done during the trimester program formed the basis for two papers (both currently still in progress): the first on quantum Springer theory (joint also with Monica Vazirani), and the second on skein modules (joint also with Pavel Safronov, who was one of the visitors during the program).
- Sam Gunningham worked on revisions to the articles [Gun18, Gun17], and had a number of useful conversations with David Jordan on these papers during the program. He also started preparing an article on twisted quantum Hamiltonian reduction and cuspidal D-modules, which is still in preparation.
- Iva Halacheva worked with Martina Balagovic (a member of one of the other groups of the trimester program) and Catharina Stroppel (University of Bonn, and program organizer) on the affine VW supercategory, related to the representation theory of the periplectic Lie superalgebra. In their preprint [BDEA$^+$18] (joint also with Zajj Daugherty, Inna Entova-Aizenbud, Johanna Hennig, Mee Seong Im, Gail Letzter, and Vera Serganova) they prove two main results: a basis theorem for the morphism spaces of the supercategory, and an explicit description of the centre of its endomorphism algebras.

- Benjamin Harris and Tobias Weich worked together on a follow up project from their paper on wave front sets of reductive Lie group representations [HW17], making some good progress.
- Daniel Kaplan continued work on his (since published) article [Kap18], profiting particularly from discussions with José Simental Rodríguez and Julian Külshammer.
- Cris Negron and Travis Schedler finished a project on Hochschild cohomology of complex linear orbifold quotients ($\mathbb{C}^n/G$), resulting in a preprint [NS$^+$18]. They studied the algebraic structure and learned how to extend this to Gerstenhaber structures (in progress) and connected it to the Frobenius structure on orbifold cohomology and a version defined by Fantechi and Goettsche, as well as to Ruan's cohomological hyperkaehler resolution conjecture.
- Maarten van Pruijsen collaborated with Guido Pezzini, who visited the HIM during the trimester program and gave a lecture in one of the seminars. Their project concerns branching rules of reductive groups in a multiplicity free setting and its implications for orthogonal polynomials (in progress).
- José Simental Rodríguez worked with Emily Norton (MPIM Bonn) on a conjecture of Berkesch Zamaere-Griffeth-Sam that gives a explicit standard (aka BGG) resolution of a simple unitary module in category O for the rational Cherednik algebra of type A. This resulted in a preprint [BNS18].
- José Simental Rodríguez made progress on computations of induction, coinduction and restriction functors for Harish-Chandra bimodules for rational Cherednik algebras that, in particular, show that induction and coinduction don't coincide. This will be part of the paper "Harish-Chandra bimodules for type A rational Cherednik algebras" in preparation.
- Travis Schedler worked with Gwyn Bellamy (program visitor and workshop speaker) on their program concerning symplectic resolutions. Their interactions during the trimester program contributed to two papers [BS19, BS16]. (The second preprint was already posted when the HIM program began, but during the program it was realized the need to make several corrections. It will soon be split into a couple of papers and submitted to journals.)
- Travis Schedler also acknowledges some influential discussions during the trimester program with Yanki Lekili (workshop speaker), Benjamin Gammage (HIM visitor and graduate student workshop participant), Daniel Kaplan (PhD student in the group), and David Jordan. These discussions eventually contributed to the articles: [KS19, ST18]. He also had very productive discussions with Brent Pym, that have led to current work in progress with Maatviichuk and Pym, probably to be posted in 2019.

- Yannick Bonthonneau visited Tobias Weich twice for a week during which time they were finalizing their work on Ruelle Resonance for manifolds with hyperbolic cusps [BW17].
- Tobias Weich also worked with his student Julia Budde, who visited during the trimester program. Budde is also working on wavefront sets of Lie group representations and profited from discussing with Benjamin Harris.
- Michael Wong continued work on his (successful) PhD thesis on Hochschild cohomology of matrix factorizations appearing as noncommuative mirrors of Riemann surfaces, benefiting particularly from conversations with visitors Raf Bocklandt and Yanki Lekili; he received an ERC fellowship with Ed Segal as a result of his work.
- Yaping Yang and Gufang Zhao worked on their project on double loop Grassmannians via fusions, which is still in progress, joint with Ivan Mirkovic. They had conversations with Olivier Schiffmann on the relations of cohomological Hall algebras and Yangians and with Allen Knutson about a conjecture of Kamnitzer and Knutson. They proved a simplified version of this conjecture in our joint paper with Ivan Mirkovic [MYZ18].They also had some conversations with Jose Simental about his project on Geiss-Leclerc-Schröer quiver varieties, which we found inspiring. They initiated a project studying cohomological Hall algebras and their representations of the generalized preprojective algebras, on which they made progress afterwards.
- Changjian Su visited Gufang Zhao for a few days during the program. Their discussions were integrated into the preprint [SZZ19].

5 Other comments

- The group members particularly appreciated the opportunity to work on collaborative projects with other group members, trimester participants, invited visitors, and from the wider mathematical community in Bonn. As well as continuing existing projects, several new collaborations and connections were formed over the course of the semester (for example, Peter Crooks and Maarten van Pruijssen). The excellent working conditions and funding for visitors afforded by the trimester program greatly facilitated such collaborations.
- In addition to formal collaborations, the group members cited innnumerable informal research discussions over the course of the trimester program. The friendly social environment at Poppelsdorfer Allee 82 and 45, and the daily tea and cake breaks contributed to a vibrant and energetic working atmosphere.
- Many of the group members found the lecture series on Coulomb branches to have been particularly useful, in particular the talks of Michael McBreen and Peng Shan.

Other lecture series, workshop, and seminar talks that the group found particularly useful during the course of the program include those of Gwyn Bellamy, Thomas Gobet, Eugene Gorsky, Allen Knutson, Yoshiki Oshima, Jan Schröer, Wolfgang Soergel, Catharina Stroppel, and Michel Van den Bergh.

- The group would also like to show their appreciation for the excellent staff at the HIM. Their capable handling of the administrative tasks, contributed significantly to the research productivity of the group. The daily tea and cake breaks were also particularly appreciated!

References

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