Computer aided solution of the invariance equation for two-variable Gini means

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(Joint work with Zsolt Páles)

Our aim is to solve the so-called invariance equation in the class of two-variable Gini means $\{G_{p,q} : p, q \in \mathbb{R}\}$, i.e., to find necessary and sufficient conditions on the 6 parameters a, b, c, d, p, q such that the identity

$$G_{p,q}\big(G_{a,b}(x,y), G_{c,d}(x,y)\big) = G_{p,q}(x,y) \qquad (x, y \in \mathbb{R}_+)$$

be valid. We recall that, for $p \neq q$, the Gini mean $G_{p,q}$ is defined by

$$G_{p,q}(x,y) := \left(\frac{x^p + y^p}{x^q + y^q}\right)^{\frac{1}{p-q}} \qquad (x, y \in \mathbb{R}_+).$$

The proof uses the computer algebra system Maple V Release 9 to compute a Taylor expansion up to 12th order, which enables us to describe all the cases of the equality.

Regularity results for generalized convex functions

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One of the classical results of the regularity theory of convex functions is the theorem of F. Bernstein and G. Doetsch [1] which states that if a real valued Jensen-convex function defined on an open interval I is bounded from above on a subinterval of I then it is continuous. According to a related result by W. Sierpiński [3], the Lebesgue measurability of a Jensen-convex function implies its continuity, too.
In this talk we generalize the theorems above for (M, N)-convex functions, calling a function $f : I \to J$ (M, N)-convex if it satisfies the inequality

$$f(M(x,y)) \leq N_{x,y}(f(x), f(y))$$

for all $x, y \in I$, where I and J are open intervals, M is a mean on I and $N_{x,y}$ is a suitable mean on J for every $x, y \in I$ (c.f., e.g., [2])

References

[1] F. Bernstein and G. Doetsch, *Zur Theorie der konvexen Funktionen*, Math. Ann. **76** (1915), 514–526.
[2] C. Niculescu and L.-E. Persson, *Convex Functions and Their Applications*, CMS Books in Mathematics, Springer, 2006.
[3] W. Sierpiński, *Sur les fonctions convexes mesurables*, Fund. Math. **1** (1920), 125-128.

Stability problems in the theory of information

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'... The fact that information can be measured is, by now generally accepted. How and with what expressions of measure this should be done, is an not an open and shut question, however.' – as Alfréd Rényi wrote in 1960.

However, nowadays it is known that the characterization problem of entropies is already a shut problem. Since, requiring some simple, natural and possible weak conditions about information measures, their characterization problem can be transformed into solving functional equations. For instance, the characterization problem of all recursive, semi–symmetric information measures leads to parametric fundamental equation of equation.

In my talk I should speak about the stability of this equation. The stability theory of functional equations basically deals with the following question: When it is true that the solution of an equation differing slightly from a given one, must of necessity be close to the solution of the given equation. In case of a positive answer to the previous problem, we say that the equation in question is stable.

As we will see the parametric fundamental equation of information can be stable, superstable and also hyperstable, depending on the value of the parameter. This result will be applied to prove that the system of all recursive semi–symmetric information measures is stable, as well.

Functional equations related to characterization problems of probability theory

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Functional equations have many interesting applications in characterization problems of probability theory.

In my talk I should speak about characterizations of univariate distributions, for example we will see characterizations for the gamma, normal and beta distributions by the help of functional equations. Then characterization of joint distributions by means of conditional distributions will also be considered.