Final Report: Real Trace Methods

This is the final report for the group *Real trace methods* at the Junior Trimester Program (JTP) at the Hausdorff Institute for Mathematics (HIM) in the fall of 2016. The members of or group were Emanuele Dotto (Sep. 5 - Dec. 22), Amalie Høgenhaven (Nov. 7 - Nov. 28), Kristian J. Moi (Sep. 5 - Dec. 15), Irakli Patchkoria (Sep. 21 - Dec. 22) and Sune P. Reeh (Sep. 5 - Dec. 22).

Composition of the group and research goals

Our research group resulted from a combination of two separate applications to the HIM, one by E. Dotto, A. Høgenhaven, K. Moi and I. Patchkoria to work on real algebraic K-theory and trace methods, and one by S. Reeh to work on fusion systems and transfers maps in stable homotopy theory. This joining of forces was very successful, broadening the perspective of the group and creating an atmosphere full of ideas and discussions.

Broadly speaking, the research goal for the group was to study questions arising in number theory, group theory and differential topology with methods of stable homotopy theory. One powerful method of this kind is algebraic K-theory and trace maps. Recent work of Hesselholt and Madsen [HM15] has made the formidable tools of equivariant stable homotopy theory available to study the involution on algebraic K-theory. A goal for our group was to extend these ideas far enough to start making explicit calculations. Another goal was to understand p-completion and loop transfer maps for classifying spaces of finite groups.

Activities

Our group organized a number of activities over the course of the trimester. We also benefited greatly from working with the other groups, especially the group *New perspectives in A-theory* whose interests overlapped significantly with ours.

Seminars and visitors

Together with the group *New perspectives in A-theory* our group organized a semi-weekly *K-theory seminar*. We invited Heng Xie from the University of Warwick to come on October 11-13 and had many visitors in connection with our two workshops. Our group also regularly attended the *A-theory group seminar* and worked with visitors to the *A*-theory group such as Nat Stapleton, Michael Weiss and Wolfgang Steimle.

Workshop: Hermitian K-theory and trace methods (November 7-11)

The group organized a workshop on interactions between algebraic and Hermitian K-theory with topological Hochschild homology. It consisted of individual talks as well as four mini-courses, given by Bjørn Dundas, Lars Hesselholt, Oliver Röndigs and Marco Schlichting. The mini-course of Hesselholt was given under *Felix Klein lectures* umbrella with title *Around topological Hochschild homology*, and continued into the following week. To make the event more accessible for junior mathematicians in need of travel funding the group received financial support from the SPP 1786

grant and in addition some participants were funded by the Felix Klein lecture grant. The workshop had 77 registered participants, this number includes the guests at the Junior Trimester Program.

Workshop: Fusion systems and equivariant algebraic topology (November 21-24)

S. Reeh organized this workshop on the role of fusion systems in algebraic topology. It consisted of individual lectures as well as two lecture series by Radu Stancu and Jesper Grodal. The individual talks, most of them given by young researchers, covered a range of topics involving fusion systems in topology, including homology calculations, group actions on spheres and chromatic homotopy theory. The workshop had 44 registered participants, this number includes the guests at the Junior Trimester Program.

Projects and papers

Below are some projects and papers that members of the group worked on during the JTP.

Project: On real THH This on-going joint project of E. Dotto, K. Moi, I. Patchkoria and S. Reeh, was one of the focal points of our group at the trimester program. We prove the equivalence of two models for real topological Hochschild homology (THR) and derive a formula for its $\mathbb{Z}/2$-geometric fixed points. Using the formula we compute the homotopy types of $\mathrm{THR}(\mathbb{F}_p)$ and of the geometric fixed points of $\mathrm{THR}(\mathbb{Z})$.

Paper: Comparing cyclotomic structures on different models for topological Hochschild homology [DMP+17] This joint paper of E. Dotto, C. Malkiewich, I. Patchkoria, S Sagave and C. Woo, proves the equivalence of the cyclotomic spectra arising from the Bökstedt construction of THH and the cyclic bar construction in orthogonal spectra.

Paper: Hermitian K-theory of Mackey functors and a reformulation of the Novikov conjecture [DO17] This paper of E. Dotto and C. Ogle introduces *Hermitian Mackey functors* and their real algebraic K-theory. Building on the results of the *On real THH*-project it gives a reformulation of the Novikov conjecture in terms of the trace map to THR.

Paper: A formula for p-completion by way of the Segal conjecture [RSS17] This joint paper of S. Reeh, T. Schlank and N. Stapleton describes the p-completion functor on stable maps between classifying space purely in terms of fusion data and Burnside modules.

Project (started at HIM): Real algebraic K-theory and L-theory This project of E. Dotto, K. Moi and T. Nikolaus defines real algebraic K-theory of stable ∞-categories with quadratic functor and studies its connection to the L-theory of such categories as recently defined by Jacob Lurie.

Project (started at HIM): A homotopy coherent approach to fusion systems This project of C. Barwick, S. Reeh and R. Molinier attempts to construct a good ∞-categorical model for the orbit category of an abstract saturated fusion system.

References

[DMP+17] Emanuele Dotto, Cary Malkiewich, Irakli Patchkoria, Steffen Sagave, and Calvin Woo, *Comparing cyclotomic structures on different models for topological hochschild homology*, 2017, arXiv:1707.07862.

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[HM15] Lars Hesselholt and Ib Madsen, *Real algebraic K-theory*, Available online: http://www.math.ku.dk/ larsh/papers/s05/, 2015.

[RSS17] Sune Precht Reeh, Tomer M. Schlank, and Nathaniel Stapleton, *A formula for p-completion by way of the segal conjecture*, 2017, arXiv:1704.00271.