

New Perspectives in A -theory

Final report

The *New perspectives on A -theory* group participated in the first half of the Junior Trimester Program at the Hausdorff Institute for Mathematics (HIM) in Bonn, during the fall of 2016. The members of our group were Cary Malkiewich (SUNY Binghamton, UIUC at the time), Mona Merling (Johns Hopkins) and George Raptis (Regensburg). One of our intended group members, John Lind (Reed College) was not able to attend, but he very much influenced our research focuses as we worked together on the proposal for the program. We are very thankful to the HIM for giving us the opportunity to work in such a stimulating place and offering us outstanding working conditions. Being able to interact with other groups (in particular the *Real Trace Methods* group), members of the Bonn mathematical community and visitors to the HIM during the course of our stay has led to progress in our projects and to new collaborations.

Overview of our group's research focus

Waldhausen's algebraic K -theory of spaces $A(X)$ is an algebraic invariant of topological spaces with deep geometric content. When X is a smooth manifold, $A(X)$ contains not only the stable homotopy type of X but also the stable pseudo-isotopy theory of X . At the same time, since $A(X) = K(S[\Omega X])$ is a special case of the algebraic K -theory of ring spectra, Waldhausen's K -theory also contains arithmetic information and is useful in the study of the K -theory of ordinary rings.

The two main focuses of our group were (1) to understand the relationship between the A -theory transfer, the transfer on topological Hochschild homology and the Becker-Gottlieb transfer, and (2) to generalize A -theory to the case when the input space has a group action on it.

Activities

We organized a seminar with invited speakers and an internal joint seminar together with the the *Real Trace Methods* group in order to facilitate interaction between the two groups. Both seminars have been very well attended and gave us and the other participants the opportunity to learn about new developments related to our topics of interest and also to interact with other experts. Most of the invited speakers spent a whole week at the HIM, which gave us the chance to have in depth discussions and start new collaborations.

Resulting projects, papers, and collaborations

Some collaborative projects that we worked on at the HIM, and some discussions that we hope will continue and lead to full-fledged projects:

- A joint project of C. Malkiewich, M. Merling and G. Raptis that was started at the HIM and was our main focus as a group is aimed at understanding the A -theory transfer maps for perfect fibrations $E \rightarrow B$, in terms of the pieces of the splitting $A(X) \simeq \Sigma_+^\infty X \times Wh(X)$. We discussed this project with members of the JPT program at the HIM, in particular with M. Ullman, and with M. Weiss and O. Sommer from U. Münster, who were invited speakers in our seminar. One of our group members, C. Malkiewich, has visited the HIM again in the summer of 2017 and has had the opportunity to discuss this project with W. Lück, which reinvigorated our efforts.
- Part of an ongoing joint project of M. Merling and C. Malkiewich on equivariant A -theory, was completed at the HIM. The resulting paper

C. Malkiewich and M. Merling, *Equivariant A-theory*, [arXiv:1609.03429](#)

gives two definitions of equivariant A -theory, one related to bivariant A -theory and one related to equivariant h -cobordisms. A follow-up paper will prove an equivariant version of the stable parametrized h -cobordism theorem.

- G. Raptis and W. Steimle have worked during Steimle’s visit to the HIM as an invited speaker in our seminar on a project on giving a “cobordism model” for Waldhausen’s S_\bullet -construction. A paper on this is forthcoming.
- C. Malkiewich started a collaboration at the HIM on comparing the cyclotomic spectra arising from the Bökstedt construction of THH and the cyclic bar construction in orthogonal spectra, which led to the paper
E. Dotto, C. Malkiewich, I. Patchkoria, S Sagave and C. Woo, *Comparing cyclotomic structures on different models for topological Hochschild homology*, [arXiv:1707.07862](#).
- While N. Stapleton and J. Noel were visiting the HIM as invited speaker in our seminar, they discussed with C. Malkiewich and M. Merling a project on connecting of a spectral version of the character map from representation theory that Malkiewich and Merling were thinking about with the Hopkins-Kuhn-Ravenel character and Stapleton’s transchromatic character map.
- E. Dotto, M. Merling and K. Moi have discussed a project started by Dotto and Merling on proving an equivariant version of Waldhausen’s additivity theorem. Discussions with I. Patchkoria on this subject have been very illuminating.

Taking advantage of the wonderful working conditions at the HIM and the opportunity to discuss ideas with the other members of the program, our group members have also individually worked on the following projects and papers (some with collaborators who were not at the HIM at the time):

- C. Malkiewich was working remotely with K. Ponto on a project on building a connection between homotopy theory and dynamics. A paper

C. Malkiewich and K. Ponto, *Periodic orbits and topological restriction homology*

is in progress, which will set the stage for this new direction in algebraic topology.

- G. Raptis was working on the final stages of the manuscript

G. Biedermann, G. Raptis and M. Stelzer, *The realization space of an unstable coalgebra*, to appear in *Astérisque*.

- G. Raptis worked on a devissage theorem for Waldhausen K -theory and discussed some aspects of his work with members of the program. A paper by Raptis is forthcoming.

- M. Merling was working on finishing the paper

J.P. May, M. Merling and A. Osorno, *Equivariant infinite loop space theory, I. The space level story*, [arXiv:1704.03413](https://arxiv.org/abs/1704.03413)

on equivariant infinite loop space theory with collaborators who were not at the HIM. She had the opportunity to discuss this project with S. Schwede and learn his invaluable insights, which led to another paper about to be posted

B. Guillou, J.P. May, M. Merling and A. Osorno, *Equivariant infinite loop space theory II. The multiplicative Segal machine*

- M. Merling also worked on finishing the paper

A. Beaudry, K. Hess, M. Kedziorek, M. Merling and V. Stojanoska, *Motivic homotopical Galois extensions*, to appear in *Topology and its Applications*, [arXiv:1611.00382](https://arxiv.org/abs/1611.00382).

on motivic Galois extensions with collaborators who were not at the HIM was in its final stage while I was there. Being able to discuss certain aspects of it with participants at the HIM was very helpful.