

Junior Trimester Program “Topology”

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**Working group “ $\ell^2$ -invariants”**

Final report

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Our group was concerned with topological and group theoretic areas where  $\ell^2$ -invariants have proven to be useful tools. In the course of the trimester, we were reading about twisted  $\ell^2$ -torsion invariants for 3-manifolds, cost of measurable equivalence relations and approximation of  $\ell^2$ -Betti numbers and rank gradient under the Farber condition. As research projects, we mainly discussed the two following problems:

- The approximation of Novikov–Shubin invariants by finite dimensional counterparts;
- The profinite invariance of  $\ell^2$ -invariants, in particular  $\ell^2$ -torsion.

For the first of these we wanted to study whether, in a sequence of finite groups “approximating” an infinite discrete group  $G$ , the rate of decay of spectral gaps (in terms of the order of the groups) are related to the Novikov–Shubin invariants. This is related to determinant approximation in the sense of Lück and in the only case where the latter is known to hold (for  $G = \mathbb{Z}$ ) one of us [1] proved such a result, which was one of our motivations. We managed to get a rudimentary result in another approximation setting used by Li–Thom (we did find a much more elementary proof of one of their results). We think there might be more interesting work to be done on this, and that there might be possible (though not straightforward) relations to Lück’s original problem.

The motivation for the second question is that for 3-manifolds,  $\ell^2$ -torsion is proportional to volume and the question has gained interest recently whether two 3-manifolds have the same volume if their fundamental groups have isomorphic profinite completions. One idea we had in this context is to ask if volume is a profinite invariant for  $S$ -arithmetic subgroups in (possibly differing) semisimple algebraic groups of higher rank. The advantage in this setting is that for most higher rank  $S$ -arithmetic groups the congruence subgroup property is known to hold true. As a consequence, the profinite completion of an  $S$ -arithmetic group gives information on the  $k_v$ -points of the surrounding algebraic group for finite places  $v$ . This could potentially allow the

conclusion of equal volume employing the Prasad volume formula. We are still working on this question and plan to publish some partial results in the near future [2]. A complete answer is still not in reach due to many technical difficulties, in particular so in the arising Bruhat-Tits theory. Some more articles have benefited from the program [3, 4].

In October we had our workshop entitled “New directions in  $L^2$ -invariants” with international guests and interesting talks on recent developments. It was a successful event to which we have obtained positive and grateful feedback from the participants. Another regular activity was our “ $L^2$ -seminar” with both local and non-local speakers which in addition to the HIM participants has also attracted researchers from both the MPI and the Mathematical Institute.

We also benefited from the activities of the other groups. For example, some of us attended various sessions of the Farrell–Jones seminar and of the 4-manifold group. The daily afternoon meetings were a welcome opportunity to keep in touch with the other groups.

Administrative staff was extremely well-organized, efficient and helpful before, during and after the program. The Hausdorff Institute has provided all necessary infrastructure and a stimulating atmosphere for the program. The junior version of the trimester programs is particularly enjoyable as for most of us it was the first opportunity to carry out an independent research program on our own responsibility. We all agree that we enjoyed the event as a whole and hope to be back at HIM at later stages of our career.

#### REFERENCES

- [1] H. Kammeyer, *Approximating Novikov–Shubin invariants of virtually cyclic coverings*, Groups Geom. Dyn. (to appear).
- [2] ———, *Profinite commensurability of  $S$ -arithmetic groups* (to appear).
- [3] S. Kionke, *Characters,  $L^2$ -Betti numbers and an equivariant approximation theorem* (2017), eprint. arXiv: 1702.02599.
- [4] J. Raimbault, H. Baik, and B. Petri, *Subgroup growth of right-angles Artin and Coxeter groups* (to appear).