From September to December 2016, as part of the Junior Topology Trimester at the Hausdorff Institute for Mathematics, we participated in a research subgroup of the larger activity focused on low-dimensional topology. The participants of our group were: Christopher Davis, Peter Feller, Min Hoon Kim, Duncan McCoy, Jeffrey Meier, Allison Miller, Jung Hwan Park, Mark Powell, Patrick Orson, Matthias Nagel, and Arunima Ray.

Amongst all manifolds, those of dimension 4 remain among the least understood. Our research group studied 4-manifolds and related topics, especially the embeddings of spheres and discs inside 4-manifolds: the closely related theories of knot and link concordance, and 2-knots. We also took the opportunity to learn more about key tools in the study of 4-manifolds.

Our group was based in four offices in the HIM annex, which allowed frequent interactions and a vibrant and exciting atmosphere. We also met formally every Monday afternoon between 3-4pm to present new problems, progress on current problems, or to let the group know what research was planned for the week. The discussions sparked by these sessions led to several of the new projects outlined below.

The high quality coffee and cake, and the opportunity this provided to meet and interact with other young mathematicians from many of the different aspects of topology, was much appreciated. Likewise, the weekly joint seminar, in which a group leader explained the background to the research of his or her group, was extremely instructive.

Learning seminars. We ran four learning seminars during the semester, two during September and October and two in November and December. The goal of these was to increase the knowledge of the group in subject areas in which certain members have a particular speciality. A lot of recent work in the field uses Heegaard Floer homology and other gauge theory techniques, and all the members of our group make use of these tools. However some topological techniques were less well known to recent graduate students in the field, and this was rectified by some of the learning seminars. Moreover we learnt about trisections of 4-manifolds, which is a fruitful new method for describing and manipulating smooth 4-manifolds. Here are our learning seminars and their organisers:

(1) Filtrations of knot concordance and signature obstructions (CD, AM, MHK, AR).
(2) Trisections of 4-manifolds (JM).
(3) Surgery theory and applications to 4-manifolds and knot theory (PO and MP).
(4) Freedman's disc embedding theorem (MP and PF).

The Freedman learning seminar gave us the occasion to revisit lecture notes from the 2013 MPIM 4-manifolds semester, and rewrite large portions of it, producing a new version of the proof of Freedman's theorem. This should be a very useful service to the community. All of our group participated in this rewriting process, each assigned a chapter to improve. We also re-watched and discussed the videos from the 2013 lecture series as part of the learning seminar.

In addition, we attended learning seminars of the Farrell-Jones conjectures group, and the conference of the $L^{(2)}$-invariants group, which were the two closest research groups to ours.

Conference. We organised, with the help of the MPIM, a hugely successful conference on 4-manifolds and knot concordance, with many of the leading figures in the field (ourselves excluded) giving talks. Much new interaction was engendered.

Visitors. The following people visited us for between a week and a month and contributed to the programme through collaborations, discussions, giving seminars and guest lecture series: Paolo Aceto, Jae Choon Cha, Matt Hedden, Shelly Harvey, Ana Lecuona, Jonathon Hillman, Lukas Lewark, Andrew Lobb, Stefan Friedl, Alex Zupan, Juanita Pinzon Caicedo, Kyungbae Park, Stefan Behrens, Boldizsar Kalmar, Marco Golla.

New mathematics. Here are some of the scientific outcomes of this semester.

(1) CD–MN–PO–MP: used a refinement of Milnor's triple linking numbers to determine precisely when two links admit homeomorphic surface systems. The article is very near completion.
(2) CD: studied the solvable filtration and links in homology spheres. There is a natural extension of the solvable filtration of Cochran-Orr-Teichner to the setting of knots and links in homology spheres. The main result is that for every $n \in \mathbb{N}$, every link in a homology sphere is n-solve-equivalent to a knot in S^3. The article is very near completion.
(3) CD–MN–JHP–AR: studied smooth and topological concordance of knots in $S^1 \times S^2$. They showed that there is a unique smooth concordance class of knots with winding number one. Additionally, they demonstrated the distinction between the smooth and topological concordance of knots in $S^1 \times S^2$. A paper written, listed below.
(4) CD–JHP–AR: studied the effect of cabling operators on the n–solvable filtration of the knot concordance group and related this to the failure of Kauffman's conjecture on slice knots. They also studied a new filtration, the handlebody filtration, which arises naturally while studying the failure of Kauffman's conjecture. This work is in progress.
(5) DM: proved that a torus knot has only finitely many non-integer non-characterizing slopes. A paper written, listed below.
(6) DM and Lewark: calculated many new values of the smooth and topological slice genera for knots with 11 and 12 crossings. This subsumed previous work by DM. A paper written, listed below.
(7) PF and Lewark: studied slice genera of knots using the Seifert form. A paper written, listed below.
(8) MHK and Cha: showed that the graded quotient of the bipolar filtration $\{\mathcal{T}_n\}_{n=0}^{\infty}$ has infinite rank at every stage greater than one. The article is very near completion.
(9) MHK–JHP and Krcatovich: constructed infinitely many 2-component links with unknotted components which are topologically concordant to the Hopf link, but not smoothly concordant to any 2-component link with trivial Alexander polynomial. A paper written, listed below.
(10) MHK and Hedden and K. Park: found irreducible 3-manifolds whose fundamental groups have weight one, but they cannot be obtained by doing 0-surgery on a knot in S^3. These manifolds are not homology cobordant to any Seifert-fibred manifold. The article is very near completion.
(11) AM and Picirillo: proved that there exist non smoothly concordant knots with diffeomorphic zero traces. Proved that there exist invertible satellite maps on the smooth concordance group which do not act by connected sum with any fixed knot. A paper written, listed below.
(12) PF–MP: proved a criterion that implies boundary links are weakly topologically slice, making use of the topological surgery learnt in the seminars. Weakly slice links are the boundary of some locally flat planar surface in the 4-ball. They are working on extending it to other classes of links.
(13) AM–MP: computed metabelian Blanchfield forms and used this to provide a new computable slice obstruction for satellite knots. It also simplifies old proofs that certain knots are not slice. The article is very near completion.
(14) JM–MP: described a diagrammatic calculus of banded links in 4–manifolds, which represents embedded surfaces. It is based on a diagrammatic calculus due to Akbulut. In order to prove the fundamental results, we need embedded Cerf theory, which is the subject of another ongoing project of Powell with Borodzik and Teichner.
(15) JM and Zupan: studied trisections of 4-manifolds, and the related theory of bridge trisections of surfaces, which provides presentations of knotted surfaces in 4-manifolds. They related trisections to the generalized property R conjecture. Two papers, listed below.
(16) MN–PO–MP, with Friedl and with JHP: studied the problem of almost concordance of knots in 3–manifolds, which is concordance modulo local knots. They discovered new

obstructions and produced knots to realise those obstructions. They also exhibited the difference in the smooth and topological categories in this setting. Two papers written, listed below.

(17) MP, with Kasprowski–Teichner: continued work on a project to classify 4-manifolds up to stable diffeomorphism, which means allowing connected sum with $S^2 \times S^2$ or with $\mathbb{CP}^2$. A great deal of progress was made during the trimester, in particular on expanding the class of fundamental groups. The article is near completion.

Completed papers that arose from the trimester.

(1) Concordance of knots in $S^1 \times S^2$, by Christopher W. Davis, Matthias Nagel, JungHwan Park and Arunima Ray. arXiv:1707.04542.
(2) On classical upper bounds for slice genera, by Peter Feller and Lukas Lewark. arXiv:1611.02679.
(3) Satellites and concordance of knots in 3-manifolds, by Stefan Friedl, Matthias Nagel, Patrick Orson and Mark Powell. To appear in Transactions of the AMS. arXiv:1611.09114.
(4) Links with nontrivial Alexander polynomial which are topologically concordant to the Hopf link, by Min Hoon Kim, David Krcatovich and JungHwan Park. arXiv:1703.10325.
(5) Non-integer characterizing slopes for torus knots, by Duncan McCoy. arXiv:1610.03283.
(6) Characterizing Dehn surgeries on links via trisections, by Jeffrey Meier and Alexander Zupan. arXiv:1707.08955
(7) Trisections and spun 4–manifolds, by Jeffrey Meier. arXiv:1708.01214
(8) Knot traces and concordance, by Allison N. Miller and Lisa Piccirillo. arXiv:1702.03974.
(9) Smooth and topological almost concordance, by Matthias Nagel, Patrick Orson, Jung Hwan Park and Mark Powell. arXiv:1707.01147.
(10) The round handle problem, by Min Hoon Kim, Mark Powell and Peter Teichner. arXiv:1706.09571.
(11) On calculating the slice genera of 11- and 12-crossing knots, by Lukas Lewark and Duncan McCoy. To appear in Experimental Mathematics. arXiv:1508.01098. (This paper was uploaded to the arXiv as a replacement of an unpublished note by the second author. The version from the trimester appeared in November 2016.)