

HIM JUNIOR TRIMESTER TOPOLOGY 2016 FINAL REPORT

Interactions between Goodwillie calculus, chromatic methods, and unstable homotopy theory

Our research group, consisting of Rosona Eldred, Gijs Heuts, Akhil Mathew, and Lennart Meier, worked at the Hausdorff Institute in October and November 2016. Our aim was to understand the implications of recent developments in Goodwillie calculus for unstable homotopy theory. More specifically, we studied unstable v_n -periodic homotopy theory, which is a localization of homotopy theory focusing on certain periodic phenomena in homotopy groups. The case $n = 0$ corresponds to rational homotopy theory. For $n > 0$, we pursued models of these v_n -periodic homotopy theories related to Lie algebras and commutative coalgebras in the category of spectra. These models are analogous to those of Quillen and Sullivan for rational homotopy theory.

Seminars. During our stay we organized the ‘Seminar on functor calculus and chromatic methods’, with research talks by Lukas Brantner, Dustin Clausen, Wolfgang Lück, Tomer Schlank, and Sarah Yeakel. Moreover, Greg Arone gave a very successful series of three lectures on various applications of Goodwillie calculus to homotopy theory, including chromatic homotopy theory. Other than giving great talks, each of our visitors provided valuable input to our research projects.

Research. The work we did during our time at the HIM has sparked a collaboration that is still ongoing. So far this has resulted in the following:

- (1) In the recent preprint [1], we show that the Bousfield-Kuhn functor is part of a monadic adjunction, which shows that v_n -periodic unstable homotopy theory can be described as a homotopy theory of algebras for a monad on the category of $T(n)$ -local spectra.
- (2) The soon to appear [2] will show that the monad of (1) is in fact the free Lie algebra monad, giving a model for v_n -periodic unstable homotopy theory generalizing Quillen’s Lie algebra model for rational homotopy theory.
- (3) Mathew proved a useful nilpotence lemma which allows for the interchange of certain limits with Goodwillie’s n -excisive approximation functor P_n in the setting of telescopic homotopy theory. Dually, his results allow the interchange of certain colimits with the dual approximations P^n studied by McCarthy. These results are also used in [2].
- (4) In rational homotopy, any loop space is automatically an infinite loop space. Bousfield proved an analogue of this statement in v_1 -periodic homotopy theory for spaces with finite p -exponent, which Meier generalized to a statement applying to v_n -periodic homotopy theory for any n .
- (5) We have an ongoing research project on costabilization of homotopy theories, which intends to give a general framework in which the Bousfield-Kuhn functor can be characterized as the costabilization of v_n -periodic unstable homotopy theory. These methods should also be useful in the context of homotopy theories of algebras (and coalgebras) over operads (resp. cooperads).

REFERENCES

- [1] Eldred, R, and Heuts, G, and Mathew, A, and Meier, L. "Monadicity of the Bousfield-Kuhn functor", arXiv:1707.05986.
- [2] Heuts, G. "Lie algebras and v_n -periodic spaces", to appear.