

# JUNIOR TRIMESTER PROGRAM “NEW TRENDS IN REPRESENTATION THEORY”

SEPTEMBER 1 - DECEMBER 18, 2020

**Group:** Telescope-type problems in representation theory

**Research areas:** torsion pairs in module categories and their derived categories, finite-dimensional algebras, silting theory, generalised telescope conjecture

**Group members:** Rosanna Laking, Frederik Marks and Jorge Vitória

**Group leader:** Rosanna Laking

**Overview:** The members of the research project attended the Junior Trimester Program on *New Trends in Representation Theory* from 28 September 2020 until 18 December 2020. Despite the restrictions made necessary by the COVID-19 pandemic, the Hausdorff Institute for Mathematics was able to provide an excellent atmosphere for research including small group discussions and online seminars.

The theme of the research project was to investigate questions inspired by the famous Telescope Conjecture in the context of the representation theory of finite-dimensional algebras. In particular we aimed to understand whether certain torsion pairs (t-structures, co-t-structures and stable t-structures) in discrete derived categories of finite-dimensional algebras are determined by “small” objects.

Our work on this project was structured around regular group discussions and benefited from biweekly seminars whose speakers and topics were chosen to be closely related to our ongoing work. As well as these activities directly related to our project, we also attended online lectures as part of the Winter School “Connections between representation theory and geometry” and the Felix Klein Lectures 2020 on “Quiver moduli and applications” given by Markus Reineke (Bochum). We are very grateful to the Hausdorff Institute of Mathematics for facilitating a very positive research environment, despite the adverse circumstances.

**Research outcomes:** During the Junior Trimester Program we were able to answer a strong version of the central question in our project proposal. That is, we were able to show that discrete derived categories of finite-dimensional algebras satisfy the strongest possible telescope property: all torsion pairs with definable torsion-free class are generated by compact objects. We had expected to use the tools of Silting Theory to show such a result, but instead we could show that

this property is related to the fact that a certain dimension on these categories (Krull-Gabriel dimension) is defined.

Following on from this result, we began to investigate properties of t-structures in bounded derived categories with defined Krull-Gabriel dimension. We were able to establish strong structural results about the hearts of these t-structures its realisation as a quotient of a functor category such as the existence of simple objects and the fact their Krull-Gabriel dimension is bounded by that of the ambient category.

As such, we may ask whether we can axiomatise collections of objects that yield Krull-Gabriel filtrations of the hearts of such t-structures. In particular, in the case of Krull-Gabriel dimension zero, these collections should generalise the well-known simple-minded collections. We are currently working to develop these results into a paper that we expect to be completed in 2022.

**Interactions with other groups:** Even though the participants of the program were unable to meet all together during our time in Bonn, we had the opportunity to meet together with other participants in smaller groups. Rosanna Laking met with two groups to discuss connections between Bridgeland stability conditions/scattering diagrams and “large objects” in derived categories of finite-dimensional algebras: firstly with Jenny August, Matthew Pressland and Hipolito Treffinger and secondly together with Mikail Gorsky and Alexandra Zvonareva. The discussions with the latter group have led to an ongoing joint project. She also had stimulating discussions together with Ilke Canakci, Martin Kalck and Matthew Pressland on the topic of infinite rank cluster categories. Frederik Marks additionally engaged into productive mathematical discussions with Alexandra Zvonareva and Martin Kalck.

**Seminar:** Together with the group working on non-commutative motives, we organised an online research seminar. Despite the very specific nature of the series of talks, we had a reasonable attendance record, not only among HIM participants, but also from mathematicians in other institutions (we had approximately 15 participants in each talk). Our research group invited the following speakers to speak about topics related to our project:

- Mike Prest (Manchester), *Krull-Gabriel dimension,  $m$ -dimension and Cantor-Bendixson rank in the Ziegler spectrum.*
- Jan Stovicek (Charles University, Prague), *The smashing spectrum of a valuation domain.*
- Lidia Angeleri Hügel (University of Verona), *Silting complexes over hereditary rings.*
- Michal Hrbek (Czech Academy of Sciences in Prague), *Telescope conjecture and its variants in derived categories of commutative rings.*