

Final Report

Junior Trimester Program

New Trends in Representation Theory

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- **Group name:** Higher Auslander–Reiten theory and tau-tilting theory.
- **Group members:** Jenny August, Johanne Haugland, Karin M. Jacobsen, Sondre Kvamme, Yann Palu, Hipolito Treffinger.
- **Duration:** September 1 - December 18, 2020.

We would like to thank the staff at the Hausdorff Research Institute for Mathematics (HIM) for their great help and support during the programme. We know that a lot of hard work went into providing us with such excellent working conditions, particularly during a pandemic. This was greatly appreciated, enabling all the members of the group to have fruitful scientific interactions with each other and with other participants of the programme.

1 Scientific activities

One of the highlights of our trimester programme was the winter school we organised in collaboration with the group *Cluster algebras and algebras from surfaces*. This was titled ‘Connections between representation theory and geometry’ and was motivated by recent results linking gentle algebras with Fukaya categories from symplectic topology. We brought together experts to explain this connection through a series of mini-courses:

- **James Pascaleff:** An introduction to Fukaya categories;
- **Bernhard Keller:** Introduction to A -infinity structures;
- **Sibylle Schroll:** A geometric model for the bounded derived category of a gentle algebra.

These mini-courses were then complemented by a collection of talks giving the state-of-the-art research in this direction.

The challenging circumstances of the pandemic meant that the whole event had to be held online. We were worried this would mean valuable networking opportunities would be lost but the HIM very kindly funded the use of the online social platform ‘Gather’ to counteract this. The online nature of the conference also had its advantages as we could reach a much wider audience, with over 150 attendees at some of the talks.

We would also like to particularly thank Stefan Hartmann, without whom this school could not have happened. He was extremely helpful and efficient in solving all our organisational problems.

2 Research Projects

- **All the members of the group** started a new project on generalizing tau-tilting theory to higher Auslander–Reiten theory. This combined in a nice way the expertise of some of the group members on higher Auslander–Reiten theory with the expertise of some of the other group members on tau-tilting theory. We met most Fridays from 09:00 to 12:00 during the programme to work on this project, and we have continued the collaboration after the programme finished. Currently we are in the process of writing up our results [AHJKPT].

This project would not have been possible without the junior trimester programme, which enabled all the members to meet in person and work together for a long period of time.

- **Jenny August** worked on generalising the connection between type A cluster combinatorics and type A singularity theory to an infinite setting. This is joint work with Man-Wai Cheung, Eleonore Faber, Sira Gratz and Sibylle Schroll, but it was significantly helped by Jenny and Sira being able to meet in-person during the programme. This is currently in the final stages of being written up [ACFGS].
- **Johanne Haugland** finished the preprint [HS] (joint with Mads H. Sandøy) while supported by the HIM, which establishes a notion of higher Koszul duality and relates this to n -hereditary algebras.
- **Johanne Haugland** also worked on a project with Raphael Bennett-Tennenhaus, Mads H. Sandøy and Amit Shah aiming to describe and study structure-preserving functors in higher homological algebra. The first results arising from this collaboration are expected to appear in the preprint [BTHSS] soon.
- **Johanne Haugland and Karin Jacobsen** had a project looking for examples of d -cluster-tilting subcategories in the module category or derived category of a gentle algebra. Together with Sibylle Schroll, they have now proved that there are no such examples aside from ones already known in the literature. This has resulted in the preprint [HJS].
- **Karin Jacobsen** started a new collaboration during the programme with Job D. Rock where they have been combining their interests of gentle algebras and continuous quivers. This is still ongoing, but so far they have results extending the ideas of relations and admissible ideals to the continuous setting.
- **Sondre Kvamme** was able to finish a project during the programme, which led to the publication [HKR].
- **Sondre Kvamme and Yann Palu** worked on the theory of exact ∞ -categories and their derived categories (joint with Gustavo Jasso and Tashi Walde). This was done by finding a new connection between the notion of a category of fibrant objects in homotopy theory, and the notion of a coresolving subcategory in representation theory. The work was facilitated by the fact that Gustavo Jasso was also in Bonn as a participant of the programme.
- **Yann Palu** took the opportunity of his stay during the semester to work with Mikhael Grosky on two ongoing projects in collaboration with Hiroyuki Nakaoka. The first one concerns higher positive and negative extensions in extriangulated categories and led to the preprint [GNP]. The second project is almost completely written down and a preprint should be on arXiv soon. It is related to mutation of rigid objects in hereditary extriangulated categories, a setting that encompasses mutation of cluster tilting objects, mutation

of 2-term silting objects, as well as some combinatorial flips appearing in relation to gentle algebras.

- **Hipolito Treffinger** started a collaboration with fellow trimester participant Severin Barmier on the algebraic phenomena arising in the amplituhedron, which led to the preprint [BOPRT] (joint with Prafulla Oak, Aritra Pal and Koushik Ray). This collaboration started from a chance meeting over coffee at the HIM so truly would not have happened without the trimester programme.
- **Hipolito Treffinger** finished a previous collaboration studying the connection between classical torsion pairs and their higher counterparts. The resulting publication [AJST] (joint with Javad Asadollahi, Peter Jørgensen, and Sibylle Schroll) was the basis for the research project carried out by our group during the trimester.
- **Hipolito Treffinger** gave a series of lectures at the LMS Autumn Algebra School while he was supported by the HIM. This also resulted in a survey article [T] which will appear in the proceedings of the school, and which will be published by Cambridge University Press.
- **Hipolito Treffinger** also started another collaboration while supported by the HIM on deformation theory for finite cluster complexes. This project (joint with Nathan Ilten and Alfredo Nájera Chávez) led to the preprint [INT].

References

- [AJST] Javad Asadollahi, Peter Jørgensen, Sibylle Schroll and Hipolito Treffinger, *On higher torsion classes*, to appear in Nagoya Mathematical Journal, [arXiv:2101.01402](#).
- [ACFGS] Jenny August, Man-Wai Cheung, Eleonore Faber, Sira Gratz and Sibylle Schroll, *Cluster structures for the A_∞ curve singularity*, in preparation.
- [AHJKPT] Jenny August, Johanne Haugland, Karin M. Jacobsen, Sondre Kvamme, Yann Palu and Hipolito Treffinger, *τ_n -tilting theory and higher torsion classes*, in preparation.
- [BOPRT] Severin Barmier, Prafulla Oak, Aritra Pal, Koushik Ray and Hipolito Treffinger, *Towards a categorification of scattering amplitudes*, [arXiv:2112.14288](#).
- [BTHSS] Raphael Bennett-Tennenhaus, Johanne Haugland, Mads H. Sandøy, Amit Shah, *The category of extensions and a characterisation of n -exangulated functors*, in preparation.
- [GNP] Mikhail Gorsky, Hiroyuki Nakaoka and Yann Palu, *Positive and negative extensions in extriangulated categories*, [arXiv:2103.12482](#).
- [HJS] Johanne Haugland, Karin M. Jacobsen and Sibylle Schroll, *The role of gentle algebras in higher homological algebra*, [arXiv:2107.01045](#).
- [HS] Johanne Haugland, Mads H. Sandøy, *Higher Koszul duality and connections with n -hereditary algebras*, [arXiv:2101.12743](#).
- [HKR] Ruben Henrard, Sondre Kvamme, and Adam-Christiaan van Roosmalen, *Auslander's formula and correspondence for exact categories*, Accepted for publication in *Advances in Mathematics* (2022) [arXiv:2011.15107](#), pages 1-36.

[INT] Nathan Ilten, Alfredo Nájera Chávez, and Hipolito Treffinger, *Deformation Theory for Finite Cluster Complexes*, arXiv:2111.02566.

[T] Hipolito Treffinger, *τ -tilting theory - an introduction*, to appear in Proceedings of the LMS Autumn Algebra School, CUP, arXiv:2106.00426.