Intertemporal Surplus Management with Jump Risks

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ABSTRACT

I have developed an intertemporal portfolio choice model with jump risks. It can be applied to pension and life insurance funds, and private investors. Following the model of Rudolf and Ziemba (2004), these long-term investors aspire to "maximize the intertemporal expected utility of the surplus", which is defined as "assets net of liabilities". Return on liabilities are modelled by a typical pure-diffusion process. Return on assets are assumed to follow a jump-diffusion process with two jump components. More specifically, the first jump component represents a systemic risk according to Das and Uppal (2004) and the second jump component represents an idiosyncratic risk according to Jarrow and Rosenfeld (1984). An investor's optimal portfolio consists of three funds: a market portfolio, a liability-hedging portfolio, and a riskless asset. In contrast to the results of Rudolf and Ziemba (2004), a market portfolio not only hedges diffusion risk, but it also hedges systemic risk and it takes into account idiosyncratic jump risk so that the investor is additionally protected against both a systemic risk and an idiosyncratic jump risk.

Keywords: Asset; Asset Allocation; Contagion; Funding ratio; HARA utility function; Hedge portfolio; Intertemporal capital asset pricing model; J-function; Jump-diffusion processes; Liabilities; Log utility function; Surplus management

JEL classification: G11, G23

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1 Introduction

Risk management for private investors is of current interest.[2] For example, the European central bank benefits of analyses of newly collected financial data of European households to measure the robustness of the financial system or the effects of monetary policy. According to the results of such analyses, individuals have been increasing their risk constantly by taking more debt for years. Especially the strong and positive correlation between the fortune of an individual and its indebtedness attracts one's attention. Comparably, well-off households are known to invest their assets more aggressively and participate in equity markets more often.[3] Some private debtors (and their creditors) underestimate the hazard of a high level of financial obligations, that can cause financial distress if the economic situation deteriorates. For instance, the countries of Western Europe and the USA reveal a not as yet known high level of consumer insolvencies in 2010.[4]

In general, individuals do not match their assets and liabilities properly. More specifically, individuals are exposed to biometric, family, political, and economical risks and often fail to manage these risks correctly because this task demands too much of them.[5] Several developments of society and politics require individuals to provide to a greater extent for the future. For example, politicians have been reacting to demographic changes[6] and have been realizing various reforms in industrialized countries for years. As a result many benefits of welfare

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[2] Similar to the Federal Reserve Board, that has been publishing financial and consumptive information (i. e. data of debt, financial obligations, and income) of US citizens for many years, the European Central Bank has established a Household Finance and Consumption Network in 2006. that aims at collecting comparable data about European households regularly.

This paragraph draws upon the European Central Bank (2009), p. 5, p. 16, and p. 25, the Federal Reserve Board (2012), p. 1 ff., and Datamonitor (2006), p. 23 ff.

[3] See Campbell (2006), p. 1569.

[4] 384,895 citizens of European countries (please note that there are no data available for some European countries), and 1,573,000 citizens of the USA have filed for consumer insolvency in 2010. See Creditreform (2011), p. 8 and p. 40.

[5] See Börsch-Supan (2005), p. 449 ff.

[6] The key factors are a rising longevity and a diminishing gross reproduction rate. See Rudolf (2004), p. 106.

states have disappeared.[7] Similarly, the traditional role of the family, that is supposed to care for its members by protecting them from any risk, diminishes.[8] The public and private pension provisions have to be regarded as insufficiently secured in several industrialized countries because of current uncertainties.[9] For example, the high indebtedness of many industrialized nations has been causing a still persisting sovereign crisis since 2010, that leads rating agencies to both have a closer look at the creditworthiness of several states and downgrade some of them. Thus, the question arises whether the affected nations will be able to carry on with their public retirement system or whether difficulties to finance the usual pension provisions will emerge in the future. At the same time individuals fail to save a sufficient amount of money during the phase of employment although they are aware of the necessity to partially finance the phase of retirement.[10] In order to explain the imperfect savings behaviour of human beings either psychological phenomena can be consulted[11] or potential misuses of established tools can be analyzed or shortcomings of employed methods can be evaluated.[12] A strong demand for risk management services, that address individuals´ needs, can be deduced.

In 2007 a global crisis began, that disclosed among other things significant weaknesses up to severe failures of risk management, that some providers of private banking services had offered.[13] The industry is being aware of the positive impact of high-quality risk management on reputation for some years.[14] According to a global survey, 28 percent of private banking providers regard reputation risk as essential and 13 percent even estimate it to be one of the greatest threats to their business.[15] In accordance with these judgements, more

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7 See Soteriou and Zenios (2003), p. 3.

8 See Börsch-Supan (2005), p. 450 f.

9 See Bertocchi, Schwartz, and Ziemba (2010), p. 3.

10 See for example Creditreform (2010), p. 18, and Ziemba (2003), p. 95 f.

11 See von Nitzsch and Hackethal (2005), p. 11.

12 See Elnekave (2007), p. 91.

13 See PricewaterhouseCoopers (2009), p. 44.

14 See PricewaterhouseCoopers (2005), p. 17.

15 See PricewaterhouseCoopers (2004), p. 5.

than a quarter of interviewed high net worth individuals (HNWIs) has withdrawn assets from their provider or entirely changed it in 2008 whereas the other part of the surveyed HNWI remained loyal to their respective provider because of their risk management and due diligence services.[16]

As a result of the loss of assets under management, the private banking industry needs to respond to HNWIs´ revealed preferences for high-quality risk management and currently discusses asset and liability management as an auspicious approach. In general, Asian and American providers have been using higher-quality risk management tools than European providers for years.[17] Thus, the European financial service industry for HNWIs has begun to catch up with developing stochastic multi-period asset allocation models.[18] However, these newly launched approaches still have the weakness of neglecting liabilities. 86 percent of interviewed private banking providers, whose majority practices its profession in Europe, perceived the advantages of asset and liability management while the quality of their prevailing risk management tools was regarded as low in 2008.[19] However, 55 percent of these surveyed managers have doubted whether the introduction of an asset and liability management tool could achieve an added value. They especially estimated the complexity of the approach as the main challenge by considering the qualification of their respective team.

Individuals (including HNWIs) strongly demand comprehensive asset and liability management models.[20] The majority of these tools is still developed for institutional investors.[21] Since the end of the nineteen-nineties, some attention has been shifting to the possibilities of modelling assets and liabilities for individuals because well-known experts had estimated the potential market to be huge.[22] In

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16 See Capgemini and Merrill Lynch (2009), p. 20 and 27.

17 See PricewaterhouseCoopers (2003), p. 18, and Reiche (2005), p. 1 ff.

18 See Amenc, Martellini, Milhau and Ziemann (2009), p. 100, and Häger and Raffelsberger (2005), p. 28 ff.

19 See Amenc, Goltz, and Schröder (2009), p. 47.

20 See Consiglio, Cocco, and Zenios (2007), p. 184.

21 See Consiglio, Cocco and Zenios (2007), p. 167 f.

22 See Mulvey and Ziemba (1999), p. 18.

general, individuals are confronted with the fact that only a few well-functioning, high-quality tools are designed for their needs, as for example “Financial Engines” for the US retirement market and an online system for the Italian banking market.[23] Most of the asset and liability management models for individuals exclude illiquid assets.[24] The “Home Account Advisor” and the model of Bertocchi, Schwartz, and Ziemba (2010, p. 370) take into account real estate and can be regarded as an exception.[25] Furthermore, research disregarded private banking until the beginning of the early 2000s.[26]

The aim of this paper is the integration of two jump components into an intertemporal surplus management model. Following Rudolf and Ziemba (2004), the construction can be applied to pension and life insurance funds. Furthermore, the paper has the common methodical procedure to conceptually transfer the institutional asset and liability management approach to individuals´ needs.[27] As a result, the developed model can as well be applied to private investors (including HNWIs).[28] The obtained liability-driven investment (LDI) technique especially satisfies the needs of wealthy individuals.[29] More specifically, the model solution reflects that rich households should pursue a risk averse behaviour because their possibilities to finance future goals strongly depend on the positive results of their investment strategy.[30]

I protect an investor from the impact of systemic risk on an optimal portfolio choice by integrating the first jump component into an intertemporal surplus management model. There exist two methodologies in the literature to take account of market shocks and financial crises: Either researchers use the extreme

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23 See Financial Engines (2012), p. 1 ff., and Consiglio, Cocco, and Zenios (2004), p. 287.

24 See Consigli (2007), p. 761.

25 See Berger and Mulvey (1999), p. 653.

26 See Schaubach (2003), p. 4.

27 See Mulvey (2005), p. 1.

28 See Consiglio, Cocco and Zenios (2007), p. 171, and Höcht, Kah Hwa, Rösch and Zagst (2008), p. 30.

29 A liability-driven investment technique not only considers performance by selecting a respective portfolio but also takes into account risk management by choosing a liability hedging portfolio. See Amenc, Goltz, and Schröder (2009), p. 43 f.

30 See Campbell, and Viceira (2002), p. 220, and Ziemba (2003), p. 72.

value approach or they apply jump-diffusion models.[31] This paper uses the second one (i.e. I develop a jump-diffusion model). Furthermore, there are two approaches in the literature to model contagion or systemic risk: Either experts develop regime switching models or they resort to integrate joint poisson jumps.[32] I take account of systemic risk by using the second approach. Research projects have found empirical evidence for the existence of jumps in returns of international equities.[33] According to Das and Uppal (2004), I define systemic risk as rare incidents, that are characterized by a high correlation across many assets.[34] For example, the correlation between assets has approached one during the systemic crisis in 2008.[35] Thus, this definition is plausible and of current interest.

Mr. Jean-Claude Trichet, who is the former President of the European Central Bank and the former Chairman of the European Systemic Risk Board, has warned on October 11, 2011, that the current [sovereign] crisis was of a systemic dimension and had leaped over from smaller to larger EU countries already.[36] A historical perspective illustrates that the crisis, that began with a financial and economical crisis in the years 2007 until 2009, cannot be seen as an exception but as a regularly returning incident.[37] Thus, such rare events should generally be considered by financial models. Various reasons for the rise of systemic risk in international financial markets exist. These causes comprise the impact of hedge funds, macroeconomic imbalances, that are unrecognized by political leaders, the

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[31] See Consigli (2002), p. 1357.

[32] See Branger, Kraft and Meinerding (2009), p. 95. In contrast to the definition of systemic risk, the definition of contagion risk does not imply an assumption about the correlation of assets.

[33] See for example the literature mentioned by Das and Uppal (2004) on page 2809 and their own important empirical outcome.

[34] I am aware that the International Monetary Fund, the Bank of International Settlement, and the Financial Stability Board use another definition of systemic risk. See, Rudolf (2012), p. 4. I regard my definition as correct in the given context because I comply with the common approach to integrate systemic risk into a portfolio selection model.

[35] See Amenc and Martellini (2011), p. 1.

[36] See Handelsblatt (2011), p. 5.

[37] See Rudolf (2010), p. 2.

increased complexity of various financial products, and spill-over effects from large companies to small ones.[38]

By integrating a second jump component into the intertemporal surplus management model, I take account of idiosyncratic jump risk. I define this type of risk as an unexpected shock of single assets´ returns. According to the literature, there is no general agreement about the widely discussed matter whether a price has to be paid for idiosyncratic risk.[39] This paper assumes according to the CAPM-based literature that the respective shock is diversifiable in the market portfolio and that the investor does not receive any risk premium for idiosyncratic jump risk.[40] Although the investor does not obtain any risk premium for this type of risk there is an impact of it on the optimal portfolio selection strategy.[41] Some portfolio selection models with idiosyncratic jumps were developed.[42] However, these approaches neither take liabilities nor systemic risk into account.

According to the results of an empirical analysis, it is reasonable to assume that the shocks of returns of small companies´ stock are diversifiable.[43] Idiosyncratic shocks can be caused by information, that concern a single company or industry only, as for example the discovery of an oil field or the proceedings of a lawsuit.[44]

Liu, Longstaff, and Pan (2003) find out that individuals with illiquid assets (as for instance real estate) invest similarly to those persons that use a jump-diffusion model for their investment strategy. The research results of Mizrach (2008) confirm the assumption that the Case-Shiller Repeat Housing Price index

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[38] See Farrell, Lund, Gerlemann, and Seeburger (2007), p. 16, and 40, International Monetary Fund (2009), p. 1, Capgemini and Merrill Lynch (2009), p. 7, and Conrad, Gultekin, and Kaul (1991), p. 597.

[39] See Huang, Liu, Rhee, and Zhang (2010), p. 147.

[40] See Jarrow and Rosenfeld (1984), p. 337 f. and p. 342.

[41] See Merton (1976), p. 134, and Jarrow and Rosenfeld (1984), p. 338 ff.

[42] See Aase (1984), p. 81, and Jeanblanc-Picqué and Pontier (1990), p. 287.

[43] See Conrad, Gultekin, and Kaul (1991), p. 597 ff.

[44] See Merton (1976), p. 133.

reveals significant jumps, that are mainly caused by information. The price and volatility process of real estate can be distorted due to the particular valuation procedure of this asset class.[45] Thus, the second jump component of my intertemporal surplus management approach can be used to integrate real estate risk and potentially further illiquid asset classes into a portfolio selection model.

Due to the rise of oil investors there was a global increase of liquidity that has inflationary effects on some asset classes (i. e. especially illiquid ones are affected) besides some positive implications.[46] As a result there is a need for new techniques to get a deeper understanding of such a development. The model of this paper can incorporate real estate risk, which is among other things a significant risk for private banking customers.[47]

In Section 2 I develop an intertemporal portfolio selection model with jump risks and obtain a three-fund theorem. Following Rudolf and Ziemba (2004), I introduce risk tolerance coefficients to the model and analyze their relationship to the funding ratio in Section 3. In Section 4 I conclude the paper. Derivations can be found in the appendixes.

2 An intertemporal surplus management model with jump risks – a three-fund theorem

Following Rudolf and Ziemba (2004) I develop an intertemporal surplus management model. This approach is suitable not only for pension funds and life insurance funds but also meets the requirements of private investors.[48] Models designed for pension funds are especially feasible to be conceptually transferred to satisfy private investors' needs.[49] The target groups aim at achieving the

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45 See Mulvey (2002), p. 174.

46 See Farrell, Lund, Gerlemann, and Seeburger (2007), p. 17, and 47,

47 See Amenc, Martellini, Milhau, and Ziemann (2009), p. 115 ff.

48 See Consiglio, Cocco and Zenios (2007), p. 171.

49 See Amenc, Martellini, Milhau and Ziemann (2009), p. 101.

maximum of the expected lifetime utility. In particular, I develop a strategy that optimizes the surplus in such a way that it balances the current and the future surplus against one another.[50] I make a contribution to the existing literature by integrating jump risks into an intertemporal surplus management model. I proceed similar to Das and Uppal (2004) by comparing a jump-diffusion model with a pure-diffusion model.

First of all, I assume that the liabilities, $L(t)$, and the assets, $A(t)$, follow conventional stochastic processes for $t \geq 0$:

$$dL(t) = L(t)\left[\mu_L dt + \sigma_L dZ_L(t)\right], \tag{1}$$

$$dA(t) = A(t)\left[\hat{\mu}_A dt + \hat{\sigma}_A dZ_A(t)\right], \tag{2}$$

with

$$E_t\left[\frac{dA(t)}{A(t)}\right] = \hat{\mu}_A dt, \tag{3}$$

$$E_t\left[\frac{dA_i(t)}{A_i(t)} \times \frac{dA_j(t)}{A_j(t)}\right] = \hat{\sigma}_{ij} dt = \hat{\sigma}_i \hat{\sigma}_j \hat{\rho}_{ij} dt, \quad i = 1, \ldots, n, \; i \neq j, \tag{4}$$

where $dA(t)/A(t)$ is the return process of the assets, $A_i(t)$ denotes the price of asset i, n is the entire number of risky assets, that an investor can choose for her portfolio, and the correlation between the Wiener processes $dZ_i(t)$ and $dZ_j(t)$ is defined as $\hat{\rho}_{ij} dt = E_t\left[dZ_i(t) \times dZ_j(t)\right]$.[51] Further, I assume a correlation between the Wiener processes $dZ_A(t)$ and $dZ_L(t)$, and denote this mathematical property by $\hat{\rho}_{AL} dt = E_t\left[dZ_A(t) \times dZ_L(t)\right]$. I specify parameters of the pure-diffusion process for

[50] See Campbell and Viceira (2002), p. 123.

[51] See Das and Uppal (2004), p. 2812, and Rudolf and Ziemba (2004), p. 977 and p. 987.

the assets, and other quantities related to the pure-diffusion model, with a ^ (carat) over the variable.[52]

I enlarge the specified process for the assets in equation (2) by adding two jump components according to Jarrow and Rosenfeld (1984) and Merton (1976).

Following Das and Uppal (2004), the first jump component represents systemic risk and is restricted in two ways:[53] First of all, I assume a simultaneous jump (i.e. of the total number of risky assets) for the processes of return. Secondly, I suppose a perfect correlation for the jump size if such a seldom jump occurs. In contrast to Das and Uppal's model, I thirdly assume a common jump size for all assets according to Jarrow and Rosenfeld and weight the jump size by an asset specific risk factor, $\mathrm{s_A}$, as shown in equation (5) below. I present the mathematical conditions for $\mathrm{s_A}$ in appendix I. I need the last restriction to obtain a portfolio strategy based on a three-fund theorem in contrast to the results of Das and Uppal.

The second jump component represents idiosyncratic risk, that is defined as a unexpected shocks of returns of single assets. The investor does not receive any risk premium for this jump risk because I assume it to be diversifiable in the market portfolio according to Jarrow and Rosenfeld (1984). When compared with their approach, that focuses on the development of an instantaneous capital asset pricing model with a jump-diffusion process, I develop a portfolio selection model. Following Merton there is an impact on the optimal portfolio choice strategy although the investor does not receive any premium for this risk.

Enlarging the pure-diffusion process for the assets in equation (2) by two jump components, I derive:

$$dA(t) = A(t)\left\langle \mu_A dt + \sigma_A dZ_A(t) + s_A \tilde{\pi}_A^S dX_A^S(t) + \left\{ -\lambda_A^I K_A^I dt + \tilde{\pi}_A^I dX_A^I(t) \right\} \right\rangle, \tag{5}$$

[52] See Das and Uppal (2004), p. 2812.

[53] Systemic risk implies both that the assets are strongly correlated and that correlated adjustments seldom occur whereas systematic risk only describes that the assets and a joint risk factor are correlated and does not restrict them with any further assumption. See Das and Uppal (2004), p. 2812.

where $dX_A^S(t)$ is the Poisson process of the systemic jump component with intensity λ_A^S, $\tilde{\pi}_A^S$ is the random jump amplitude of the systemic jump component, $dX_A^I(t)$ is the Poisson process of the idiosyncratic jump component with intensity λ_A^I, and $\tilde{\pi}_A^I$ is the random jump amplitude of the idiosyncratic jump component.[54] $\tilde{\pi}_A^S$ and $\tilde{\pi}_A^I$ describe how much the asset price moves in percent if a systemic or an idiosyncratic jump occurs respectively. I assume that the logarithm of $\tilde{\pi}_A^S$ is distributed normal with mean K_A^S and variance $\left\langle v_A^S \right\rangle^2$. Comparably I assume that the logarithm of $\tilde{\pi}_A^I$ is distributed normal with mean K_A^I and variance $\left\langle v_A^I \right\rangle^2$. Moreover, I assume that the Wiener processes $\mathrm{dZ_A}(\mathrm{t})$ and $\mathrm{dZ_L}(\mathrm{t})$ are correlated, which is denoted by $\rho_{\mathrm{AL}}\mathrm{dt} = \mathrm{E_t}[\mathrm{dZ_A}(\mathrm{t}) \times \mathrm{dZ_L}(\mathrm{t})]$. $\mathrm{dZ_A}(\mathrm{t})$, $dX_A^S(t)$, $dX_A^I(t)$, $\tilde{\pi}_A^S$, $\tilde{\pi}_A^I$ are assumed to be independent. $\mathrm{dZ_L}(\mathrm{t})$, $dX_A^S(t)$, $dX_A^I(t)$, $\tilde{\pi}_A^S$, $\tilde{\pi}_A^I$ are also assumed to be independent.

The total expected return of the assets, that follow the jump-diffusion process in equation (5), is described in equation (6). The investor receives a risk premium for both the diffusion risk, μ_{A}, and the systemic jump risk, μ_A^J:[55]

$$E_t\left[\frac{dA(t)}{A(t)}\right] = \mu_A dt + s_A \lambda_A^S K_A^S dt = \mu_A dt + \mu_A^J dt \,. \tag{6}$$

Further, the total covariance of the assets, that are assumed to follow the jump-diffusion process in equation (5), is described in equation (7). By calculating the co-moment I assume a perfect correlation of the systemic jump size, $\tilde{\pi}_A^S$, across the entire number of risky assets.[56] Moreover, I assume the diversification of idiosyncratic jump risk in the market portfolio so that the idiosyncratic jump size, $\tilde{\pi}_A^I$, is uncorrelated with the market.[57] More specifically, the total covariance

54 See Jarrow and Rosenfeld (1984), p. 338 and p. 342, and Das and Uppal (2004), p. 2812.

55 See Das and Uppal (2004), p. 2813.

56 See Das and Uppal (2004), p. 2813.

57 See Jarrow and Rosenfeld (1984), p. 338 and 341.

equals the sum of the covariance between the diffusion elements, σ_{ij}, and the covariance between the systemic jump elements, σ_{ij}^J:[58]

$$E_t\left[\frac{dA_i(t)}{A_i(t)} \times \frac{dA_j(t)}{A_j(t)}\right] = \sigma_i\sigma_j\rho_{ij}dt + s_i s_j \lambda_A^S\left(\left\langle K_A^S\right\rangle^2 + \left\langle v_A^S\right\rangle^2\right)dt = \sigma_{ij}dt + \sigma_{ij}^J dt. \tag{7}$$

The correlation between the Wiener processes $dZ_i(t)$ and $dZ_j(t)$ is defined as $\rho_{ij}dt = E_t\left[dZ_i(t) \times dZ_j(t)\right]$. I analyze the differences between a pure-diffusion model, whose processes for the liabilities and assets are specified in equations (1) and (2), and a jump-diffusion model, whose processes for the liabilities and assets are stated in equations (1) and (5). The latter model additionally protects an investor against systemic risk and considers idiosyncratic jump risk. In order to proceed with my analysis I adopt a proposition of Das and Uppal (2004), given in the equations (8) and (9) below, so that I select the first two moments of the assets' jump-diffusion process, that are specified in equations (6) and (7), to equal precisely the corresponding first two moments of the assets' pure-diffusion process, that are given in equations (3) and (4):[59]

$$\mu_A = \hat{\mu}_A - \mu_A^J, \tag{8}$$

$$\sigma_{ij} = \hat{\sigma}_{ij} - \sigma_{ij}^J. \tag{9}$$

The proposition implies that the first two moments of the pure-diffusion process of the assets and the jump-diffusion process of the assets are equalized. For example, I calculate the difference of the total expected return, $\hat{\mu}_A$, and μ_A^J because the systemic jump component, $s_A \tilde{\pi}_A^S dX_A^S(t)$, is going to add the latter term. Correspondingly, I can interpret the modification of the covariance. "In this way, she [the investor] reduces the expected return and covariance coming from

[58] See Das and Uppal (2004), p. 2813.

[59] See also Jarrow and Rosenfeld (1984), p. 343, and Merton (1976), p. 129 f.

the diffusion terms in order to offset exactly the contribution of the jump."[60] Despite of the equalizing proposition the models result in different portfolios because the jump-diffusion process of the assets is characterized by higher moments (see equations (22) and (23)).

Next, I state the surplus, $S(t)$, and the funding ratio, $F(t)$:[61]

$$S(t) := A(t) - L(t),$$
$$F(t) := A(t)/L(t). \tag{10}$$

Subsequently, I denote the return process for the assets, $\mathrm{R_A(t)}$, and the one for the liabilities, $\mathrm{R_L(t)}$, by:[62]

$$R_A(t) := dA(t)/A(t),$$
$$R_L(t) := dL(t)/L(t).$$

Following Sharpe and Tint (1990), Rudolf and Ziemba (2004), and according to equation (10) I state the surplus process:

$$R_S(t) = \frac{dS(t)}{A(t)} = R_A(t) - \frac{R_L(t)}{F(t)},$$
$$\begin{aligned} R_S(t) = {} & \left[\mu_A - \frac{\mu_L}{F(t)}\right]dt + \sigma_A dZ_A(t) - \frac{\sigma_L}{F(t)} dZ_L(t) + s_A \tilde{\pi}_A^S dX_A^S(t) \\ & + \left[-\lambda_A^I K_A^I dt + \tilde{\pi}_A^I dX_A^I(t)\right]. \end{aligned} \tag{11}$$

Based on $dS(t) = R_S(t)A(t)$ and applying equation (11) I derive

[60] See Das and Uppal (2004), p. 2814.

[61] See Rudolf and Ziemba (2004), p. 977.

[62] See Rudolf and Ziemba (2004), p. 978, and Das and Uppal (2004), p. 2811 f.

$$E_t\left[dS(t)\right] = A(t)\left[\mu_A - \frac{\mu_L}{F(t)}\right]dt\,,$$
$$dS(t)dS(t) = A^2(t)\left[\sigma_A^2 + \frac{\sigma_L^2}{F^2(t)} - 2\frac{\sigma_{AL}}{F(t)}\right]dt\,, \tag{12}$$

where $\sigma_{\mathrm{AL}} := \sigma_{\mathrm{A}}\sigma_{\mathrm{L}}\rho_{\mathrm{AL}}$ denotes the covariance between the assets and the liabilities.[63]

Following Gihman and Skorohod (1970, pp. 263-272), Malliaris and Brock (1990), pp. 121-124), and Piazzesi (2005) I apply $dS(t) = R_S(t)A(t)$ once again and use equation (11) so that I derive the systemic jump component of the surplus process, γ_S^S, and the idiosyncratic jump component of the surplus process, γ_S^I:

$$\gamma_S^S = A(t)s_A\tilde{\pi}_A^S\,,$$
$$\gamma_S^I = A(t)\tilde{\pi}_A^I\,. \tag{13}$$

Following Rudolf and Ziemba (2004), the investor has the subsequent optimization problem:

$$J(S,t) := \max_{\omega} E_t\left(\int_t^T U(S,\tau)\,d\tau\right),$$
$$J(S,t) = \max_{\omega} E_t\left(\int_t^{t+dt} U(S,\tau)\,d\tau + \int_{t+dt}^T U(S,\tau)\,d\tau\right), \tag{14}$$

where "U is an additively separable, twice differentiable, concave utility function", T states the end of the planning horizon of the investor, "E_{t} denotes expectation with respect to the information set at time t", the J-function is the

[63] See Rudolf and Ziemba (2004), p. 978.

maximum of the intertemporal expected utility, that an investor derives from the surplus during her lifetime, and $\boldsymbol{\omega} \in \Re^n$ represents the vector of portfolio weights of the risky-assets.

According to Kushner (1967, Chapter 4), Jarrow and Rosenfeld (1984), and Rudolf and Ziemba (2004) the optimal control satisfying equation (14) exists and can be derived by using the Bellman principle and solving:

$$J(S,t) = U(S,t)\,dt + \max_{\omega} E_t\left[J(S + dS, t + dt)\right]. \tag{15}$$

Subsequently, I apply the generalized Itô formula for jump-diffusion processes and combine it with equations (12) and (13) so that I derive (see appendix II for proof):

$$0 = U(t,S) + \max_{\omega}[\Delta J]$$

$$0 = U(t,S) + \max_{\omega}\begin{bmatrix} J_t + J_S A(t)\left(\mu_A - \frac{\mu_L}{F(t)}\right) + \frac{1}{2} J_{SS} A^2(t)\left(\sigma_A^2 + \frac{\sigma_L^2}{F^2(t)} - 2\frac{\sigma_{AL}}{F(t)}\right) \\ + \lambda_A^S E_t\left\langle J\left(S + A s_A \tilde{\pi}_A^S, t\right) - J(S,t)\right\rangle \\ + \lambda_A^I E_t\left\langle J\left(S + A\tilde{\pi}_A^I, t\right) - J(S,t)\right\rangle \\ \end{bmatrix}, \tag{16}$$

where $\mathrm{J_t}$ is the first partial derivative of J with respect to t, $\mathrm{J_S}$ represents the first partial derivative of J with respect to S, and $\mathrm{J_{SS}}$ denotes the second partial derivative of J with respect to S.[64]

I assume that $\boldsymbol{m}_A \in \Re^n$ is "the vector of expected asset returns of the risky assets", $\boldsymbol{e} \in \Re^n$ is the vector of ones, $\boldsymbol{v}_{AL} \in \Re^n$ is the vector of covariances between the n assets and the liabilities, $\boldsymbol{V} \in \Re^{n \times n}$ is the matrix of covariances between the n risky assets,[65] $\boldsymbol{s} \in \Re^n$ is the vector of asset specific risk factors, that weight the systemic jump amplitude, and $\tilde{\pi}_A^S$, $\boldsymbol{j} \in \Re^n$ is the vector of random

[64] See Rudolf and Ziemba (2004), p. 979, and Jarrow and Rosenfeld (1984), p. 341.

[65] See Rudolf and Ziemba (2004), p. 979.

jump amplitudes of the idiosyncratic jump component. Subsequently, I transform equation (16) and derive (the proof and definitions of the matrix and the vectors can be found in the appendix III):

$$0 = U(t,S) + \max_{\omega} \begin{bmatrix} J_t + J_S A(t)\left(\boldsymbol{\omega}'(\boldsymbol{m}_A - r\boldsymbol{e}) + r - \frac{\mu_L}{F(t)}\right) \\ + \frac{1}{2} J_{SS} A^2(t)\left(\boldsymbol{\omega}'\boldsymbol{V}\boldsymbol{\omega} + \frac{\sigma_L^2}{F^2(t)} - 2\frac{\boldsymbol{\omega}'\boldsymbol{v}_{AL}}{F(t)}\right) \\ + \lambda_A^S E_t\left\langle J(S + A\boldsymbol{\omega}'\boldsymbol{s}\tilde{\pi}_A^S, t) - J(S,t)\right\rangle + \lambda_A^I E_t\left\langle J(S + A\boldsymbol{\omega}'\boldsymbol{j}, t) - J(S,t)\right\rangle \end{bmatrix}, \quad (17)$$

where r denotes the return of the riskless asset.[66] Then I differentiate equation (17) with respect to $\boldsymbol{\omega}$ and obtain:[67]

$$\begin{aligned} \boldsymbol{\omega} \;=\; & -\frac{J_S}{A(t)J_{SS}}\boldsymbol{V}^{-1}(\boldsymbol{m}_A - r\boldsymbol{e}) - \frac{\lambda_A^S}{A(t)J_{SS}}\boldsymbol{V}^{-1}E_t\left\langle J_S(S + A\boldsymbol{\omega}'\boldsymbol{s}\tilde{\pi}_A^S, t)\boldsymbol{s}\tilde{\pi}_A^S\right\rangle \\ & -\frac{\lambda_A^I}{A(t)J_{SS}}\boldsymbol{V}^{-1}E_t\left\langle J_S(S + A\boldsymbol{\omega}'\boldsymbol{j}, t)\boldsymbol{j}\right\rangle + \frac{\boldsymbol{V}^{-1}\boldsymbol{v}_{AL}}{F(t)}. \end{aligned} \quad (18)$$

In order to reduce complexity, I am going to use the abbreviations $J_S(S + A\boldsymbol{\omega}'\boldsymbol{s}\tilde{\pi}_A^S, t) = J_{S(\pi)}$ and $J_S(S + A\boldsymbol{\omega}'\boldsymbol{j}, t) = J_{S(j)}$. Furthermore, I define the vector of means of the jump amplitude of the idiosyncratic jump component as $\boldsymbol{K}_I' := (K_1^I, \;\ldots\;, K_n^I)'$. Next, I state the covariance between the term $J_{S(j)}$ and the random jump amplitude of the idiosyncratic jump component of the ith risky asset, $\tilde{\pi}_i^I$, as $Cov\lfloor J_{S(j)}, \tilde{\pi}_i^I\rfloor$. I denote the vector of these covariances by $\boldsymbol{v}\lfloor J_{S(j)}, \boldsymbol{j}\rfloor' := (Cov\lfloor J_{S(j)}, \tilde{\pi}_i^I\rfloor, \;\ldots\;, Cov\lfloor J_{S(j)}, \tilde{\pi}_n^I\rfloor)$. Transforming equation (18) while using the above defined terms I derive:

66 See Rudolf and Ziemba (2004), p. 979.

67 $$\begin{aligned} 0 \;=\; & J_S(\boldsymbol{m}_A - r\boldsymbol{e}) + A(t)J_{SS}\boldsymbol{V}\boldsymbol{\omega} + \lambda_A^S E_t\left[J_S(S(t) + A(t)\boldsymbol{\omega}'\boldsymbol{s}\tilde{\pi}_A^S, t)\boldsymbol{s}\tilde{\pi}_A^S\right] \\ & + \lambda_A^I E_t\left[J_S(S(t) + A(t)\boldsymbol{\omega}'\boldsymbol{j}, t)\boldsymbol{j}\right] - \left[A(t)/F(t)\right]J_{SS}\boldsymbol{v}_{AL} \end{aligned}$$

$$\omega = -\frac{J_S}{A(t)J_{SS}}\boldsymbol{V}^{-1}(\boldsymbol{m}_A - r\boldsymbol{e}) - \frac{\lambda_A^S}{A(t)J_{SS}}\boldsymbol{V}^{-1}\boldsymbol{s}\left\{Cov\left[J_{S(\pi)},\tilde{\pi}_A^S\right] + E_t\left[J_{S(\pi)}\right]K_A^S\right\}$$
$$-\frac{\lambda_A^I}{A(t)J_{SS}}\boldsymbol{V}^{-1}\left\{\boldsymbol{v}\left[J_{S(j)},\boldsymbol{j}\right] + E_t\left[J_{S(j)}\right]\boldsymbol{K}_I\right\} + \frac{\boldsymbol{V}^{-1}\boldsymbol{v}_{AL}}{F(t)}. \tag{19}$$

In order to derive a three-fund theorem it is necessary to apply the mutual fund theorem of Ross (1978, theorem 3) to equation (19). Jarrow and Rosenfeld (1984) illustrate and prove in detail that the mutual fund theorem of Ross is applicable to jump-diffusion processes if the jump components have the particular form given in equation (5) and if ΔJ, given in equation (16), is state independent (except surplus).

In the following I explain the investor's funds i.e. the components of the optimal portfolio. The terms on the right-hand side on the first line and the left term on the second line of equation (19) represent the first component of the optimal portfolio strategy, i.e. the market portfolio. In total, the market portfolio consists of three parts. First of all, the market portfolio has the capability to hedge diffusion risk. The respective component is equal to:

$$-\frac{J_S}{A(t)J_{SS}}\boldsymbol{V}^{-1}(\boldsymbol{m}_A - r\boldsymbol{e}).$$

The market portfolio not only hedges diffusion risk but also systemic risk. The relevant second component of the market portfolio is specified by:

$$-\frac{\lambda_A^S}{A(t)J_{SS}}\boldsymbol{V}^{-1}\boldsymbol{s}\left\{Cov\left[J_{S(\pi)},\tilde{\pi}_A^S\right] + E_t\left[J_{S(\pi)}\right]K_A^S\right\},$$

where $J_{S(\pi)} = J_S\left(S + A\boldsymbol{\omega}'\boldsymbol{s}\tilde{\pi}_A^S, t\right)$ represents the above defined abbreviation. The last part of the market portfolio hedges idiosyncratic jump risk and is specified by

$$-\frac{\lambda_A^I}{A(t)J_{SS}}\boldsymbol{V}^{-1}\left\{\boldsymbol{v}\left[J_{S(j)},\boldsymbol{j}\right] + E_t\left[J_{S(j)}\right]\boldsymbol{K}_I\right\},$$

where $J_{S(j)} = J_S(S + A\omega' j, t)$ represents the above defined abbreviation. The right term on the second line of equation (19) represents the hedge portfolio for the liabilities, which is independent of the investor's preferences, and is specified by:

$$\frac{\boldsymbol{V}^{-1}\boldsymbol{v}_{AL}}{F(t)}.$$

In order to calculate the unconditional moments of the assets' return process, that is specified in equation (5), I proceed in accordance with the approach of Das and Uppal (2004). While doing so, I apply the knowledge of Curtiss (1942) about the interrelationship between the characteristic function and the moment generating function. In order to derive the moments of the continuously compounded returns of the assets i and j, I assume that $\imath = 1, \cdots, n$ and $\imath \neq \jmath$:[68]

$$\text{Mean} = t\left(\mu_i - \frac{1}{2}\sigma_i^2 + s_i \lambda_A^S K_A^S\right), \tag{20}$$

$$\text{Covariance} = t\left\{\sigma_{ij} + s_i s_j \lambda_A^S \left(\left\langle K_A^S \right\rangle^2 + \left\langle v_A^S \right\rangle^2\right)\right\}, \tag{21}$$

$$\text{coskewness} = \frac{t\lambda_A^S s_i^2 s_j K_A^S \left\{3\left\langle v_A^S \right\rangle^2 + \left\langle K_A^S \right\rangle^2\right\}}{\operatorname{var} iance_i \operatorname{var} iance_j^{1/2}}, \tag{22}$$

$$\text{excess kurtosis} = \frac{t\lambda_A^S s_i^4 \left\{3\left\langle v_A^S \right\rangle^4 + 6\left\langle K_A^S \right\rangle^2 \left\langle v_A^S \right\rangle^2 + \left\langle K_A^S \right\rangle^4\right\}}{\operatorname{var} iance_i \operatorname{var} iance_j^{1/2}}. \tag{23}$$

In order to derive the optimal control equation (19) needs to be solved numerically:

[68] See Das and Uppal (2004), p. 2819.

$$0 = J_S(\boldsymbol{m}_A - r\boldsymbol{e}) + A(t)J_{SS}\boldsymbol{V\omega} + \lambda_A^S \boldsymbol{s}\left\{Cov\left[J_{S(\pi)}, \tilde{\pi}_A^S\right] + E_t\left[J_{S(\pi)}\right]K_A^S\right\} + \lambda_A^I\left\{\boldsymbol{v}\left[J_{S(j)}, \boldsymbol{j}\right] + E_t\left[J_{S(j)}\right]\boldsymbol{K_I}\right\} - \left[A(t)/F(t)\right]J_{SS}\boldsymbol{v_{AL}} \quad (24)$$

By assuming pure-diffusion processes for the liabilities and the assets, that are specified in equation (1) and equation (2), I derive the known closed-form solution of Rudolf and Ziemba (2004):

$$\hat{\boldsymbol{\omega}} = -\frac{J_S}{A(t)J_{SS}}\hat{\boldsymbol{V}}^{-1}(\hat{\boldsymbol{m}}_A - r\boldsymbol{e}) + \frac{\hat{\boldsymbol{V}}^{-1}\hat{\boldsymbol{v}}_{AL}}{F(t)}. \quad (25)$$

where I consider the particular case of a constant investment opportunity set.[69] Similar to my proceeding the results of Das and Uppal (2004) can be compared to the closed-form solution of Merton (1971) (i. e. Das and Uppal consider systemic risk and thus enlarge the result of Merton by a jump component).

3 Risk preference, and funding ratio

I assume a utility function from the HARA class.[70] In accordance with the results of Merton (1971 and 1993, p. 140) my assumption is equivalent to the stating that the J-function (15) is part of the HARA class. I denote the risk tolerance coefficient by $\alpha < 1$. κ and θ are real constants and ρ is a utility deflator. Then

$$U(S,t) \subset HARA \Leftrightarrow J(S,t) \subset HARA,$$
$$J(S,t) = \frac{1-\alpha}{e^{\rho t}\alpha}\left(\frac{\kappa S}{1-\alpha} + \theta\right)^{\alpha}. \quad (26)$$

[69] See Jarrow and Rosenfeld (1984), p. 337.

[70] This section draws upon Rudolf and Ziemba (2004).

Equation (26) contains the implicit assumption of a linear absolute risk tolerance, which can be seen by looking at the derivation $-J_S/J_{SS} = S/(1-\alpha)+\theta/\kappa$. If I assume that α approaches $-\infty$, $\theta = 1$, and $\rho = 0$ I obtain the special case of negative exponential utility functions $J = -e^{-aS}$. If I apply the particular conditions of $\theta = 0$, $\rho = 0$, and $\kappa = (1-\alpha)^{(\alpha-1)/\alpha}$ it results isoelastic power utility $J = S^{\alpha}/\alpha$.[71] If I assume that α approaches 0 ($\theta = \rho = 0$) I obtain log utility $J = \ln(S)$, which is a special case of isoelastic power utility.

Given that I assume a utility function of the HARA class I derive the following terms:

$$J_S = \frac{\kappa}{e^{\rho t}}\left(\frac{\kappa S}{1-\alpha}+\theta\right)^{\alpha-1}, \; J_{SS} = -\frac{\kappa^2}{e^{\rho t}}\left(\frac{\kappa S}{1-\alpha}+\theta\right)^{\alpha-2},$$
$$J_S\left(S + A\boldsymbol{\omega}'\boldsymbol{s}\,\tilde{\pi}_A^S, t\right) = \frac{\kappa}{e^{\rho t}}\left(\frac{\kappa\left[S + A\boldsymbol{\omega}'\boldsymbol{s}\,\tilde{\pi}_A^S\right]}{1-\alpha}+\theta\right)^{\alpha-1}, \tag{27}$$
$$J_S(S + A\boldsymbol{\omega}'\boldsymbol{j}, t) = \frac{\kappa}{e^{\rho t}}\left(\frac{\kappa\left[S + A\boldsymbol{\omega}'\boldsymbol{j}\right]}{1-\alpha}+\theta\right)^{\alpha-1}.$$

In order to enhance the simplicity I am going to use the abbreviations $\beta\left(\tilde{\pi}_A^S\right) = \left(\frac{\kappa\left[S + A\boldsymbol{\omega}'\boldsymbol{s}\,\tilde{\pi}_A^S\right]}{1-\alpha}+\theta\right)^{\alpha-1}$, and $\beta(\boldsymbol{j}) = \left(\frac{\kappa\left[S + A\boldsymbol{\omega}'\boldsymbol{j}\right]}{1-\alpha}+\theta\right)^{\alpha-1}$. I define the covariance between the term $\beta(\boldsymbol{j})$ and the idiosyncratic jump size of asset i, $\tilde{\pi}_i^I$, as $Cov\left[\beta(\boldsymbol{j}), \tilde{\pi}_i^I\right]$. I specify the corresponding vector by $\boldsymbol{v}[\beta(\boldsymbol{j}), \boldsymbol{j}]' := \left(Cov\left[\beta(\boldsymbol{j}), \tilde{\pi}_i^I\right], \; \ldots \; , Cov\left[\beta(\boldsymbol{j}), \tilde{\pi}_n^I\right]\right)$. Subsequently, I insert the terms of equation (27) in equation (19) and obtain:

[71] See Ingersoll (1987), p. 39.

$$\boldsymbol{\omega} \quad = \quad \left\{ \frac{1}{1-\alpha}\left[1-\frac{1}{F}\right]+\frac{\theta}{A\kappa}\right\} \boldsymbol{V}^{-1}\left(\boldsymbol{m}_A - r\boldsymbol{e}\right)$$
$$+\frac{1}{A\kappa}\left(\frac{\kappa S}{1-\alpha}+\theta\right)^{2-\alpha} \boldsymbol{V}^{-1} \left\langle \begin{array}{l} \lambda_A^S \boldsymbol{s}\left\{Cov\left[\beta\left(\tilde{\pi}_A^S\right),\tilde{\pi}_A^S\right]+E_t\left[\beta\left(\tilde{\pi}_A^S\right)\right]K_A^S\right\} \\ +\lambda_A^I\left\{\boldsymbol{v}\left[\beta(\boldsymbol{j}),\boldsymbol{j}\right]+E_t\left[\beta(\boldsymbol{j})\right]\boldsymbol{K_I}\right\} \end{array} \right\rangle + \frac{\boldsymbol{V}^{-1}\boldsymbol{v}_{AL}}{F}. \tag{28}$$

The first two terms on the right-hand side of equation (28) belong to the market portfolio. The third term on the right-hand side of equation (28) is the hedge portfolio for the liabilities, which is independent of preferences.

Next, I assume the special case of log utility:

$$J(S,t)=\ln(S). \tag{29}$$

Under the assumption of log utility I obtain the following derivations:

$$\mathrm{J_S} = \mathrm{S}^{-1},\ \mathrm{J_{SS}} = -\mathrm{S}^{-2},\ J_S\left(S+A\boldsymbol{\omega' s}\tilde{\pi}_A^S,t\right)=\left(S+A\boldsymbol{\omega' s}\tilde{\pi}_A^S\right)^{-1},$$
$$J_S\left(S+A\boldsymbol{\omega' j},t\right)=\left(S+A\boldsymbol{\omega' j}\right)^{-1}. \tag{30}$$

In order to enhance simplicity I am going to use the abbreviations $\delta\left(\tilde{\pi}_A^S\right)=\left(S+A\boldsymbol{\omega' s}\tilde{\pi}_A^S\right)^{-1}$, and $\delta(\boldsymbol{j})=(S+A\boldsymbol{\omega' j})^{-1}$. I define the covariance between the term $\delta(\boldsymbol{j})$ and the idiosyncratic jump size of asset i, $\tilde{\pi}_i^I$, as $Cov\left[\delta(\boldsymbol{j}),\tilde{\pi}_i^I\right]$. I specify the corresponding vector by $\boldsymbol{v}\left[\delta(\boldsymbol{j}),\boldsymbol{j}\right]' := \left(Cov\left[\delta(\boldsymbol{j}),\tilde{\pi}_1^I\right], \ \ldots \ ,Cov\left[\delta(\boldsymbol{j}),\tilde{\pi}_n^I\right]\right)$. Subsequently, I insert the terms of equation (30) in equation (19) and obtain:

$$\boldsymbol{\omega}=\left[1-\frac{1}{F}\right]\boldsymbol{V}^{-1}\left\langle \begin{array}{l} \left(\boldsymbol{m_A}-r\boldsymbol{e}\right)+S\lambda_A^S\boldsymbol{s}\left\{Cov_t\left[\delta\left(\tilde{\pi}_A^S\right),\tilde{\pi}_A^S\right]+E_t\left[\delta\left(\tilde{\pi}_A^S\right)\right]K_A^S\right\} \\ +\lambda_A^I S\left\{\boldsymbol{v}\left[\delta(\boldsymbol{j}),\boldsymbol{j}\right]+E_t\left[\delta(\boldsymbol{j})\right]\boldsymbol{K_I}\right\} \end{array}\right\rangle + \frac{\boldsymbol{V}^{-1}\boldsymbol{v}_{AL}}{F} \tag{31}$$

Under the assumption of log utility it is possible to make an investment in the market portfolio, that is as well independent of preferences.

4 Conclusions

Individuals increasingly demand high-quality risk management services, that are tailored to their personal needs. On the one hand, households have more serious been getting into debt for years and they are at higher risk of getting into financial distress. On the other hand, individuals face the facts that they need to provide to a greater extend for the future. For example, it has become an absolute necessity to take out private retirement provisions. Asset and liability management gives private investors the benefit of allocating their assets optimally while planning the secure financing of future liabilities and goals at the same time. Thus, the development of such models for individuals is of strong current interest and further innovation in this area of research can be expected.

In 2007 an economic and financial crisis began, that had among other things an impact on the private banking industry. Providers of private banking services became aware of the added value of high-quality risk management. They witnessed that a failure of such tools damaged the reputation of the respective bank institutions as well as it induced affected HNWIs to withdraw some or all of their assets. Due to the crisis providers of private banking services learnt that high-quality risk management constitutes a key success factor. Private banking advisors perceive the benefits of asset and liability management ever since. Consequently, providers of private banking services are interested in offering advice, that is based on asset and liability management models. They want to benefit from the positive impact of these risk management tools on customer loyalty during times of crisis. However, the bank institutions recognize the complexity of these tools and are inhibited from designing and implementing them straight away.

I have responded to the strong interest, that individuals (including HNWIs) and providers of private banking services have in high-quality risk management. I have used a LDI technique, so that the model can be applied to pension and life insurance funds, and individuals (including HNWIs).

I have derived a three-fund theorem. The optimal portfolio of an investor consists of three funds: a market portfolio, a liability-hedging portfolio, and a riskless asset. In contrast to the results of Rudolf and Ziemba (2004), a market portfolio not only hedges diffusion risk, but it also hedges systemic risk and it takes into account idiosyncratic jump risk so that the investor is additionally protected against both a systemic risk and an idiosyncratic jump risk.

The 2007 to 2009 financial and economical crisis, that has turned into a sovereign crisis in 2010, illustrates how important systemic risk is to players in international financial markets. Therefore, it is of current interest to intertemporal surplus optimizers that a model takes account of systemic risk. In addition, long-term investors are attracted to alternative investments. Thus, intertemporal surplus optimizers also benefit from a model that considers idiosyncratic jump risk because it allows to take account of real estate risk among other things.

Appendix I: Derivation of the asset specific risk factor of the first jump component

The following mathematical derivation goes back to Jarrow and Rosenfeld (1984). I apply the original procedure to an intertemporal surplus management model. In particular, the jump-diffusion process of the assets, that is stated in equation (5), is split up into two parts. The first one is assumed not to be diversifiable in the market portfolio whereas the second one is supposed to be diversifiable in the market portfolio.

In the following I denote the jump-diffusion process of asset i and let n be the entire number of risky assets:

$$dA_i(t) = A_i(t)\left\langle \mu_i dt + \sigma_i dZ_i(t) + s_i \tilde{\pi}_A^S dX_A^S(t) + \left\{ -\lambda_A^I K_i^I dt + \tilde{\pi}_i^I dX_i^I(t) \right\} \right\rangle .$$

The diffusion part of asset i (dD_i) of the equation above consists of two components:

$$dD_i = \mu_i dt + \sigma_i dZ_i(t) .$$

In agreement with the arguments of Ross (1978) I deduce from the preceding equation the following mathematical relationship:[72]

$$dD_i(t) = \mu_i dt + s_i d\psi(t) + g_i d\eta_i(t) ,$$

where I assume the existence of both the constants (ϕ_i, s_i, and g_i) and the Wiener processes ($d\psi(t)$ and $d\eta_i(t)$). Moreover, the following assumptions are valid:

[72] See Jarrow and Rosenfeld (1984), p. 339.

$$s_i^2 + g_i^2 = \sigma_i^2 ,$$

$$E_t\left[d\psi(t)d\eta_i(t)\right] = 0 ,$$

$$\sum_{i=1}^{n} \phi_i = 1 ,$$

$$\sum_{i=1}^{n} \phi_i \left(g_i d\eta_i(t)\right) = 0 ,$$

$$\sum_{i=1}^{n} \phi_i \mu_i > r ,$$

where r denotes the riskless rate of interest.

It is possible to split a final number of normally distributed random variables into a common factor, $d\psi(t)$, and residuals, $d\eta_i(t)$. Both the common factor and the residuals are normally distributed. Since I use normally distributed returns I benefit from the elementary property that returns are statistically independent if their covariance is equal to zero. I assume that the Wiener processes ($d\psi(t)$ and $d\eta_i(t)$), the Poisson processes of both jump components ($dX_A^S(t)$ and $dX_i^I(t)$), and both stochastic jump amplitudes ($\tilde{\pi}_A^S$, and $\tilde{\pi}_i^I$) are independent of each other. As a result I can interpret $d\psi(t)$ and $d\eta_i(t)$ as risk factors. The first one is not diversifiable whereas the second one is diversifiable in the market portfolio. Next, I assume that the Wiener processes, $d\psi(t)$, and $dZ_L(t)$, are correlated and denote this relationship by $d\psi(t)dZ_L(t) = \rho_{\psi L}dt$. Finally, I substitute the modifications in the jump-diffusion process of asset i and I derive the following equivalent expression:

$$dA_i(t) = A_i(t)\left\langle \mu_i dt + s_i d\psi(t) + g_i d\eta_i(t) + s_i \tilde{\pi}_A^S dX_A^S(t) + \left\{-\lambda_A^I K_i^I dt + \tilde{\pi}_i^I dX_i^I(t)\right\}\right\rangle ,$$

where K_i^I denotes the expected value of the logarithm of the idiosyncratic jump amplitude, $\tilde{\pi}_i^I$. Consequently, the asset specific risk factor, s_i, belongs to the part of the jump-diffusion process in equation (5), that is not diversifiable in the market portfolio.

Appendix II: Derivation of equation (16)

In order to reduce complexity, I leave out the arguments of the J-function.[73] I define $\mathrm{o}(\mathrm{dt})$ as a variable that adds up all components of dt, whose order is higher than one. Accordingly, I have $\mathrm{o}(\mathrm{dt}): \lim_{\mathrm{dt}\to 0} \mathrm{o}(\mathrm{dt})/\mathrm{dt} = 0$. First of all, I apply the generalized Itô's formula for jump-diffusion processes:[74]

$$
\begin{aligned}
J(S,t) &= U(S,t)dt + \max_{\omega} E_t\left[J(S+dS,t+dt)\right] \\
&= Udt + \max_{\omega} E_t\left[\begin{array}{l} J + J_t dt + J_S dS + \frac{1}{2} J_{SS} dS^2 + \lambda_A^S\left\{J\left(S+\gamma_S^S,t\right) - J(S,t)\right\}dt \\ + \lambda_A^I\left\{J\left(S+\gamma_S^I,t\right) - J(S,t)\right\}dt + o(dt) \end{array}\right],
\end{aligned}
$$

$$
0 = Udt + \max_{\omega} E_t\left[\begin{array}{l} J_t dt + J_S dS + \frac{1}{2} J_{SS} dS^2 + \lambda_A^S\left\{J\left(S+\gamma_S^S,t\right) - J(S,t)\right\}dt \\ + \lambda_A^I\left\{J\left(S+\gamma_S^I,t\right) - J(S,t)\right\}dt + o(dt) \end{array}\right].
$$

Next, I modify the equation above and derive

$$
0 = Udt + \max_{\omega}\left[\begin{array}{l} J_t dt + J_S E_t(dS) + \frac{1}{2} J_{SS} dSdS + \lambda_A^S E_t\left\{J\left(S+\gamma_S^S,t\right) - J(S,t)\right\}dt \\ + \lambda_A^I E_t\left\{J\left(S+\gamma_S^I,t\right) - J(S,t)\right\}dt \end{array}\right].
$$

Finally, I substitute in the equation above equations (12) and (13) and derive equation (16).

[73] This appendix draws upon Rudolf and Ziemba (2004), p. 987.

[74] See Gihman and Skorohod (1970), p. 264, Kushner (1967), p. 18, and Malliaris and Brook (1990), p. 121 ff.

Appendix III: Derivation of equation (17)

I describe the process of the price of asset $\mathrm{A_i}(\mathrm{t})$, $\mathrm{i} = 1, \ldots, \mathrm{n}$, as:

$$dA_i(t) = A_i(t)\left\langle \mu_i dt + \sigma_i dZ_i(t) + s_i \tilde{\pi}_A^S dX_A^S(t) + \left\{ -\lambda_A^I K_i^I dt + \tilde{\pi}_i^I dX_i^I(t) \right\} \right\rangle.^{75}$$

I define the vector of the expected asset returns of the n risky assets as $\mathbf{m}_\mathbf{A}' := (\mu_1, \ \ldots \ , \mu_\mathrm{n})$. I weight the systemic jump amplitude, $\tilde{\pi}_A^S$, that is common to all assets, by a vector of asset specific risk factors, that I define as $\mathbf{s}' := (\mathrm{s}_1, \ \ldots \ , \mathrm{s}_\mathrm{n})$. Moreover, I define the vector of the random jump amplitudes of the idiosyncratic jump component as $\boldsymbol{j}' := (\tilde{\pi}_1^I, \ \ldots \ , \tilde{\pi}_n^I)$. The first part of the total covariance given by equation (7) is the covariance between the diffusion components of assets' returns, σ_{ij}. I define the corresponding matrix, that contains the covariance terms between the diffusion components, by:

$$\mathbf{V} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \vdots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix}.$$

The second part of the total covariance given by equation (7) is the covariance between the systemic jump components of assets' returns, σ_{ij}^J. I denote the resultant matrix, that contains the covariance terms between the systemic jump components, by:

[75] This appendix draws upon Rudolf and Ziemba (2004), p. 987 ff.

$$\boldsymbol{V}^J = \begin{bmatrix} \sigma_{11}^J & \dots & \sigma_{1n}^J \\ \vdots & \vdots & \vdots \\ \sigma_{n1}^J & \dots & \sigma_{nn}^J \end{bmatrix}.$$[76]

According to equation (8), I adjust the total expected return of the assets' jump-diffusion process as follows:

$$\mu_i = \hat{\mu}_i - \mu_i^J .$$

Similarly, the second moment of the assets' jump-diffusion process is modified as stated in equation (9), i.e.:

$$\sigma_{ij} = \hat{\sigma}_{ij} - \sigma_{ij}^J .$$

I assume that $x_i(t)$ represents the share of asset i, that an investor owns at the moment t. I define $B(t)$ to be equal to the amount of cash, that is invested in the asset portfolio at the instant t. Next, I derive the worth of the asset portfolio $A(t)$:

$$A(t) = \sum_{i=1}^{n} x_i(t) A_i(t) + B(t).$$

In the next infinitesimal period of time, dt, the value of the asset portfolio develops as follows:

$$dA(t) = \sum_{i=1}^{n} x_i(t) dA_i(t) + dB(t) = \sum_{i=1}^{n} x_i(t) dA_i(t) + rB(t)dt ,$$

where r represents the riskless rate of interest. I denote the vector of risky assets` portfolio weights, $\boldsymbol{\omega}' = (\omega_1, \ \dots, \ \omega_n)$, by:

[76] See Das and Uppal (2004), p. 2813.

$$\omega_i(t) := \frac{x_i(t)A_i(t)}{A(t)},\ i = 1, \ldots, n.$$

If I add up both the risky and the riskless investment the result is assumed to be one. Furthermore, I suppose that $\mathbf{e'} = (1, \ \ldots \ 1)$ represents the vector of ones, $\boldsymbol{e} \in \Re^n$. Thus, the investor holds a share of $B(t)/A(t) = 1 - \boldsymbol{\omega}'\boldsymbol{e}$ of the riskless asset. Consequently, I modify the equation above for dA and derive:

$$dA(t) = A(t)\left[\begin{array}{l} \sum_{i=1}^{n}[\omega_i(\mu_i - r) + r]dt + \sum_{i=1}^{n}\omega_i\sigma_i dZ_i(t) + \sum_{i=1}^{n}\omega_i s_i \tilde{\pi}_A^S dX_A^S(t) \\ + \left\{ -\sum_{i=1}^{n}\omega_i\lambda_A^I K_i^I dt + \sum_{i=1}^{n}\omega_i\tilde{\pi}_i^I dX_i^I(t) \right\} \end{array}\right].$$

I compare the equation above with equation (5), i.e. $dA(t) = A(t)\left\langle \mu_A dt + \sigma_A dZ_A(t) + s_A\tilde{\pi}_A^S dX_A^S(t) + \left\{-\lambda_A^I K_A^I dt + \tilde{\pi}_A^I dX_A^I(t)\right\}\right\rangle$, and obtain similar to Rudolf and Ziemba (2004):

$$\mu_A = \boldsymbol{\omega}'(\boldsymbol{m}_A - r\boldsymbol{e}) + r,$$

$$\sigma_A dZ_A(t) = \sum_{i=1}^{n}\omega_i\sigma_i dZ_i(t).$$

Subsequently, I transform the preceding equation and it results according to Rudolf and Ziemba (2004):

$$\sigma_A^2 = \boldsymbol{\omega}'\boldsymbol{V}\boldsymbol{\omega},$$

where $\mathbf{V}$ represents the previously defined covariance matrix. The covariance between the risky asset i and the liabilities is denoted by σ_{iL}. I define the corresponding vector by $\mathbf{v}'_{\mathbf{AL}} := (\sigma_{\mathrm{1L}}, \ \ldots \ , \sigma_{\mathrm{nL}})$. As a result, I obtain the relationship $\sigma_{AL} = \boldsymbol{\omega}'\mathbf{v}_{\mathbf{AL}}$.

I compare the expression above for the process of assets, $\mathrm{dA(t)}$, with equation (5), i. e. $dA(t) = A(t)\left\langle \mu_A dt + \sigma_A dZ_A(t) + s_A\tilde{\pi}_A^S dX_A^S(t) + \left\{-\lambda_A^I K_A^I dt + \tilde{\pi}_A^I dX_A^I(t)\right\}\right\rangle$, because I need to take into account both jump components as well. I obtain the following relationships for both the compounded Poisson processes of the systemic jump component and the compounded Poisson process of the idiosyncratic jump component, respectively:

$$A(t)s_A\tilde{\pi}_A^S dX_A^S(t) = A(t)\sum_{i=1}^{n}\omega_i s_i \tilde{\pi}_A^S dX_A^S(t),$$

$$A(t)\tilde{\pi}_A^I dX_A^I(t) = A(t)\sum_{i=1}^{n}\omega_i \tilde{\pi}_i^I dX_i^I(t).$$

Subsequently, I derive the following expressions for the systemic jump component of the surplus process, γ_S^S, and the idiosyncratic jump component of the surplus process, γ_S^I:

$$\gamma_S^S = A(t)s_A\tilde{\pi}_A^S = A(t)\boldsymbol{\omega}'\boldsymbol{s}\tilde{\pi}_A^S,$$

$$\gamma_S^I = A(t)\tilde{\pi}_A^I = A(t)\boldsymbol{\omega}'\boldsymbol{j}.$$

I assume a simultaneous occurrence of systemic jumps across all assets so that $dX_i^S(t) = dX_j^S(t) = dX_A^S(t)$ for all $\mathrm{i} = 1,\ldots,\mathrm{n}$, $\mathrm{j} = 1,\ldots,\mathrm{n}$ and $\mathrm{i} \neq \mathrm{j}$.[77] Consequently, the intensity of the systemic jump component, λ_A^S, is identical for all assets, i. e. $\lambda_i^S = \lambda_j^S = \lambda_A^S$. Similarly, I assume that the intensity of the idiosyncratic jump component, λ_A^I, is identical for all assets, i. e. $\lambda_i^I = \lambda_j^I = \lambda_A^I$.

I use the final expressions of expected returns, variances, covariances, jump components and intensities in order to derive equation (17).

[77] See Das and Uppal (2004), p. 2810 ff.

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