

# ON TAME DYNAMICAL SYSTEMS

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ABSTRACT. According to a dynamical version of the Bourgain-Fremlin-Talagrand dichotomy an enveloping semigroup of a dynamical system is either **tame**: has cardinality  $\leq 2^{\aleph_0}$ , or it is topologically wild and contains a copy of  $\beta\mathbb{N}$ , the Čech-Stone compactification of a discrete countable set. As a general principle one can measure the usefulness of a new mathematical notion by the number of seemingly unrelated ways by which it can be characterized. According to this principle the notion of tameness stands rather high. Tameness can also be characterized by the lack of a certain independence property — where combinatorial Ramsey type arguments take a leading role — by the fact that the elements of the enveloping semigroup of a tame system are Baire class 1 maps, and, using results of Talagrand, as systems in which the topological Glivenko-Cantelli property is satisfied. Finally a dynamical system is tame iff it can be represented on a Banach space which does not contain an isomorphic copy of  $\ell_1$ . I will review some of these results and indicate some applications.

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