# CONSISTENT PREDICTORS

MARK DAVIS, IMPERIAL COLLEGE LONDON

In risk management applications, one observes a price series $X(1), \ldots, X(n)$ and wishes to compute (based on this observation and possibly other data) some statistic relating to the conditional distribution of $X(n+1)$, such as a quantile or the CVaR (essentially, a put option value). Since no stationarity can be assumed, no subsequently observed data will ever reveal whether this calculation was correct. The only meaningful criteria relate to long-run performance. For $n = 1, 2, \ldots$, let $\pi(n+1)$ be the result of our calculation at time $n$ and let $l_n(X(1), \pi(1), \ldots, X(n), \pi(n))$ be a loss function measuring the accuracy of the prediction. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space carrying a discrete-time process $(\tilde{X}(k))$; this process is a *model for* $(X(k))$. We say that $\pi = (\pi(n)$ is a *consistent predictor* in a class $\mathcal{P}$ of probability measures if there is a loss function $l = \{l_1, l_2, \ldots\}$ such that

$$\lim_{n\to\infty} l_n(\tilde{X}(1), \pi(1), \ldots, \tilde{X}(n), \pi(n)) = 0 \quad \mathbb{P}\text{-a.s. for all } \mathbb{P} \in \mathcal{P}.$$

For quantile estimation we can establish consistent prediction where $\mathcal{P}$ is close to being the class of *all* probability measures on $(\Omega, \mathcal{F})$, using limit laws for i.i.d. random variables. For other problems such as CVaR estimation, consistency can be established in some case using limit laws for martingales. The theory here is far from complete, but will certainly involve much smaller classes $\mathcal{P}$.

This study is closely related to A.P. Dawid's theory of *prequential statistics*.