

ACTIVITY REPORT

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Main project: All currently known compact Ricci-flat manifolds have special holonomy, i.e. they are either Calabi-Yau, hyper-Kähler, G_2 , Spin(7), or flat. It is an outstanding question whether this is necessarily the case—more precisely, whether a compact, simply-connected, irreducible Ricci-flat n -manifold can have generic holonomy $SO(n)$. There seems to be little hope for a general existence theorem for the Riemannian Einstein equations that would allow one to answer such questions. On the other hand, it is sometimes possible to find compact manifolds that are *approximately* Ricci-flat, by gluing together truncations of various singular or noncompact Ricci-flat building blocks (which are typically easier to construct), such that in addition the holonomy group of these approximators is far from being special. The *singular perturbation method* in geometric analysis then suggests to apply the implicit function theorem to construct Ricci-flat holonomy $SO(n)$ metrics nearby.

Singular perturbation has been applied with great success to several elliptic geometric problems, notably the construction of self-dual Yang-Mills connections over compact 4-manifolds (Taubes) and of complete surfaces of constant mean curvature in \mathbb{R}^3 (Kapouleas). Recent work of Biquard seems to provide the first instance of a successful application of such ideas to the full Einstein equations, albeit in a noncompact setting which makes it easier to push obstructions off to infinity.

During our time at HIM we carefully studied the work of Taubes, Kapouleas and Biquard, aiming to acquaint ourselves with their techniques. However we came to realize in the end that there exist formidable obstructions to carrying out such a program for compact Ricci-flat manifolds.

Other projects:

RH completed a project with Stuart Hall (Buckingham) and Michael Siepmann (ETH Zürich) on the stability inequality for Ricci-flat cones [4]. The excellent working conditions at HIM as well as conversations with other trimester participants have been particularly helpful for this.

RH, HH and Aaron Naber (MIT) had some very fruitful discussions on quantitative stratification for geometric equations. This initiated two projects [1, 2] by RH joint with Aaron and Jeff Cheeger (Courant) on quantitative stratification for the mean curvature and harmonic map flows. Earlier in 2011 in Oberwolfach HH and Aaron had begun to study a related question (local regularity) for the Ricci flow; work on this project continued at HIM and has since resulted in the paper [7].

HH also continued a collaboration on asymptotically conical Calabi-Yau manifolds [3] with Ronan Conlon (McMaster), who was visiting the Max-Planck Institute in Bonn at the time. In particular, a talk by Frank Reidegeld (Dortmund) in the HIM seminar sparked an idea for constructing a new class of Calabi-Yau manifolds from pairs of flag varieties [3, Part I, Section 4].

Moreover, HH made some progress on a long-term project of his to do with the classification of gravitational instantons. This benefited greatly from conversations with HIM trimester participant Johannes Nordström (Imperial College London), and with Olivier Biquard and Philip Boalch (ENS Paris) who were both visiting HIM for the hyper-Kähler workshop in December.

Lectures:

In the trimester seminar, RH gave a survey talk on compactness theorems in geometry and a more specialized talk on singularities in 4d Ricci flow.

HH gave a survey talk on the geometry of compact Einstein manifolds in the trimester seminar and lectured on gravitational instantons in the hyper-Kähler workshop.

Activities organized:

RH organized, together with Panagiotis Konstantis (Tübingen) and Jan Swoboda (MPI Bonn), a workshop on geometric flows. We had 18 talks in total, including 6 talks by participants of the HIM trimester. The interaction between participants working on a variety of related topics was highly fruitful, and we very much appreciate the excellent support by the HIM.

Visitors:

RH invited Reto Müller (Imperial College London) to stay at HIM a couple of extra days around the workshop on geometric flows. We had some initial discussions on Łojasiewicz-Simon inequalities for the Ricci flow, which eventually resulted in the paper [6].

HH invited Mark Haskins (Imperial College London) to visit HIM for two weeks in December. In the course of this visit we developed an idea for a joint paper with Johannes Nordström, which has since appeared [5]. We also worked on the planning of a 2-week conference on Ricci curvature and Kähler-Einstein metrics to take place at ICMS Edinburgh this year. In addition Mark collaborated with Johannes and Diarmuid Crowley (MPI) on the differential topology of G_2 -manifolds.

REFERENCES

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