FINAL REPORT

In the period of September to December, 2011, our group, composed by the following young researchers:

Fei Han (National University of Singapore)
Alexander Kahle (Universität Göttingen)
Arturo Prat-Waldron (Max Planck Institute for Mathematics)
David Corbett Redden (Michigan State University)
Konrad Waldorf (Universität Regensburg)

was hosted by the Hausdorff Research Institute for Mathematics (HIM) in the framework of Junior Trimester Program "Differential Geometry".

HIM provided us a great environment for research. Working together in HIM, the group members can share ideas and discuss specific problems intensively. We had a Field Theory seminar, usually once a week, where new results or ideas were reported. Working in HIM also provided us opportunity to learn from experts. The visiting of Prof. Dr. Teichner to our group benefited us a lot. HIM provided financial assistance to nonlocal visitors to our group too. These visitors included Michael Joachim (Universität Münster), Thomas Nikolaus (Universität Regensburg), Dmitry Pavlov (Universität Münster) and Urs Schreiber (Universiteit Utrecht).

Our group specialized in the field theory aspects of differential geometry. Cohomology and K-theory are classical theories in differential geometry and algebraic topology. The celebrated Atiyah-Singer index theorem equals the index of the Dirac operator on a spin manifold to the $\widehat{A}$-genus, a topological invariant of the manifold. In the family case, i.e. for a family of spin manifolds and the corresponding family of Dirac operators, K-theory of the parametrizing space is the home of the family index. The Witten genus is a q-deformation of the $\widehat{A}$-genus, which is an integral modular form and can be formally thought of as the S^1-equivariant index of the Dirac operator on the free loop space of a string manifold. The theory of topological modular forms (TMF) developed by Hopkins and Miller is the home for a family of Witten genus, just as the K-theory is the home for a family of the $\widehat{A}$-genus. Unlike cohomology and K-theory, which can be described by geometric cocycles in various manners , TMF is so far only defined in the realm of homotopy theory. The Stolz-Teichner program aims to geometrize TMF by using 2 dimensional supersymmetric field theories as cocycles.

0 and 1 dimensional supersymmetric field theories have already been shown to give cohomology and K theory by Stolz, Teichner and their coauthors. Our group focused on topics related to the Stolz-Teichner program.

Differential cohomology is the refinement of the ordinary cohomology. Redden studied differential cocycles and wrote a paper about this. ("Trivializations of differential cocycles", [arXiv:1201.2919]).

As cohomology, K-theory also has smooth extension, the differential K-theory. There are several existing models for differential K-theory. As mentioned above, in the Stolz-Teichner program, K-theory has been related to 1 dimensional supersymmetric field theories. Kahle and Prat-Waldron were working on the joint project "1|1-Euclidian Field Theories and Differential K-theory" aiming to give a new model for differential K-theory with 1|1-Euclidian Field Theories over manifolds being cocycles.

In the equivariant aspects of the theory, Han jointed in a project of Prof. Dr. Teichner and Prof. Dr. Stolz, which studied the supersymmetric field theory realization of the Cartan model of equivariant cohomology as well as realization of the Bismut-Chern character as a dimension reduction functor. The relevant paper is in progress.

The Dirac operator on free loop space is not yet defined rigorously. To understand the geometry of free loop space is big challenge. The transgression map from manifold to its free loop space relates geometric objects on the original manifold to geometric objects on the free loop space. Waldorf and our visitor Nikolaus investigated along this direction, which was represented in the paper: "Lifting Problems and Transgression for Non-Abelian Gerbes", [arxiv:1112.4702].

We are indebted to HIM for the supportive working environment and the financial assistance. Our stay at HIM has benefited our research and is likely to have a lasting impact on our future research.