If the horses are allowed to bet, does it affect the odds ?

A simple model for commodities markets

Ivar Ekeland, Delphine Lautier, Bertrand Villeneuve

Chaire de Finance et Développement Durable
Université Paris-Dauphine

Bonn, May 2013

Is speculation driving the commodity prices ?

"Imagine that Joe Shmoe and Harriet Who, neither of whom has any direct involvement in the production of oil, make a bet: Joe says oil is going to $150, Harriet says it won't. What direct effect does this have on the spot price of oil — the actual price people pay to have a barrel of black gunk delivered?
The answer, surely, is none. Who cares what bets people not involved in buying or selling the stuff make? And if there are 10 million Joe Shmoes, it still doesn't make any difference.
Well, a futures contract is a bet about the future price. It has no, zero, nada direct effect on the spot price. And that's true no matter how many Joe Shmoes there are, that is, no matter how big the positions are"

Krugman, "The conscience of a liberal",. January 23, 2008

Commodity markets

- Agricultural products, energy products, non ferrous metals.
- For most of them there is a physical market and a futures market.
- On the physical market, one trades the commodity itself, either for immediate delivery (spot market) or for later delivery (forward market): this is where producers meet industrial users and consumers.
- On the futures market, one trades financial contracts (paper) : this is where hedging against a rise in the price of the underlying asset meets hedging against a decrease. This brings in new agents, who are interested not in the commodity itself, but in the risk : speculators.

Earlier work

How does the derivative market influence the physical market? What is the interplay between industrial users, and consumers on one hand, speculators and inventory holders on the other? Some of the aspects have been covered in earlier papers:

- Anderson & Danthine (1983)
- Hirshleifer (1988)
- Deaton & Laroque (1992)
- Guesnerie & Rochet (1993)

Ours is the first to study all aspects simultaneously. We introduce a very simple, perhaps the simplest possible, model, and we perform an equilibrium analysis.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. At time $t = 1$, ω_1 is observed, but $\tilde{\omega}_2$ is not.
- 2 spot markets, at $t = 1$ and $t = 2$. These are physical markets: only positive quantities can be traded.
- 1 futures market. Contracts are bought at $t = 1$ and settled at $t = 2$. This is a financial market: negative positions are allowed.
- There are 3 prices: 2 spot prices P_1 and $\tilde{P}_2$, and a futures price P_F. At time $t = 1$, P_1 and P_F are observed but $\tilde{P}_2$ is not.
- We will determine $P_1, \tilde{P}_2$ and P_F by equilibrium conditions (all markets clear) and compare market structures.

Futures for dummies

For one (unit) contract,
you pay P_F at $t = 1$ and
you receive $\tilde{P}_2$ at $t = 2$.

A negative position means that you cash P_F now and pay $\tilde{P}_2$ later

Some issues

To make things simple, interest rate is set to 0.

- Market is in contango (report) if $P_F > P_1$,
  and in backwardation (déport) if $P_F < P_1$.
  If inventory > 0, arbitrage (cash-and-carry) implies contango.
  Back'tion s'times observed with inventory > 0 (convenience yield).
- If $P_F \neq E\left[\tilde{P}_2\right]$, futures market is biased.
  Keynes argues that futures markets exhibit systematic downwards bias $P_F < E\left[\tilde{P}_2\right]$ because producers are more prone to hedge their price risk than consumers or speculators, so the latter insure the former.
- A futures contract is a bet on the commodity price, speculators do not trade the commodity, just like gamblers do not run on the horsetrack.

The agents

- **Spot traders**, who buy up anything left on the spot markets (residual demand)
- **Processors**, or industrial users, who use the commodity to produce other goods which they sell to outside consumers. Because of the inertia of the production process or because they sell their production forward, they have to decide at $t = 1$ how much to produce at $t = 2$. They cannot store the commodity, so they have to buy all of their input at $t = 2$.
- **Storers**, which have storage capacity, and who can use it to buy the commodity at $t = 1$ and release it at $t = 2$.
- **Speculators**, or money managers, who trade only in futures, not trade on the physical (spot) markets.

Rational behaviour

- All agents (except the spot traders) have mean-variance utility: if they make a profit $\tilde{\pi}$ they derive utility:

$$E[\tilde{\pi}] - \frac{1}{2}\alpha \mathrm{Var}[\tilde{\pi}], \text{ with } \alpha = \alpha_I, \alpha_P, \alpha_S$$

- They make optimal decisions at $t = 1$, based on the conditional expectation $E\left[\tilde{P}_2 \mid P_1\right]$, which will be determined in equilibrium.
- All of them (except the spot traders) take positions on the futures market, either for hedging or for speculating.

Spot traders and speculators

- **Spot traders**.
  If price at time $t = 1, 2$ is P_t, the demands from spot traders are

$$\mu_1 - mP_1 \quad \text{and} \quad \tilde{\mu}_2 - mP_2$$

- **Speculators**. The profit resulting from a futures position f_S is:

$$\pi_S(f_S) = f_S(\tilde{P}_2 - P_F)$$

$$f_S^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_S \mathrm{Var}[\tilde{P}_2]}$$

The inventory holders

Storage is costly: holding a quantity x costs $\frac{1}{2}Cx^2$. If they buy $x \geq 0$ on the spot market at $t = 1$, resell it on the spot market at $t = 2$, and take a position f_I on the futures market, the resulting profit is:

$$\pi_I(x, f_I) = x(\tilde{P}_2 - P_1) + f_I(\tilde{P}_2 - P_F) - \frac{1}{2}Cx^2$$

The optimal positions are:

$$x^* = \frac{\max\{P_F - P_1, 0\}}{C}; \qquad f_I^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_I \mathrm{Var}[\tilde{P}_2]} - x^*$$

The storer holds inventory if the futures price is higher than the current spot price.

Processors

Processors decide at time $t = 1$ how much input y to buy at $t = 2$, and which position f_P to take on the futures market. The input y results in an output $y - \frac{\beta}{2}y^2$ (decreasing returns to scale) which is sold at a price P_0. It is assumed that P_0 is known at time $t = 1$. The resulting profit is:

$$\pi_P(y, f_P) = P_0 \left(y - \frac{\beta}{2} y^2 \right) - y\tilde{P}_2 + f_P(\tilde{P}_2 - P_F)$$

The optimal positions are:

$$f_P^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_P \mathrm{Var}[\tilde{P}_2]} + y^*; \qquad y^* = \frac{\max\{P_0 - P_F, 0\}}{\beta P_0}$$

Clearing the markets

- **Futures market.** Positions can be positive or negative:

$$N_S f_S^* + N_P f_P^* + N_I f_I^* = 0$$

$$P_F = \mathrm{E}[\tilde{P}_2] + \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}} \left(N_P y^* - N_I x^* \right)$$

Clearing the markets

- **Spot market at** $t = 1$. On the supply side the harvest ω_1 and on the demand side we have the inventory $N_I x^*$ bought by the storers, and the demand of the spot traders.

$$\omega_1 = N_I x^* + \mu_1 - mP_1$$
$$P_1 = \frac{1}{m}\left(\mu_1 - \omega_1 + N_I x^*\right)$$

- **Spot market at** $t = 2$. On the supply side, the harvest $\tilde{\omega}_2$, and the inventory $N_I x^*$ sold by the storers, and, on the other side, the input $N_P y^*$ bought by the processors and the demand of the spot traders.

$$\tilde{\omega}_2 + N_I x^* = N_P y^* + \tilde{\mu}_2 - m\tilde{P}_2$$
$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - N_I x^* + N_P y^*\right)$$

The equilibrium equations

$$\text{Market Characteristic: } \rho = 1 + m \frac{\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}$$

Substituting the value for $\tilde{P}_2$ into the equations for P_1 and P_F (which are just numbers, not random variables) we get the system:

$$\begin{aligned} mP_1 - \frac{N_I}{C} \max\{P_F - P_1, 0\} &= \mu_1 - \omega_1 \\ mP_F + \rho \left(\frac{N_I}{C} \max\{P_F - P_1, 0\} - \frac{N_P}{\beta P_0} \max\{P_0 - P_F, 0\} \right) &= E[\tilde{\mu}_2 - \tilde{\omega}_2] \end{aligned}$$

which is a system of two nonlinear equations for two unknowns P_1 and P_F. If we can solve this system we derive $\tilde{P}_2$ by substituting.

Solving the equilibrium equations

We solve by investigating the piecewise linear map:

$$F\left(P_1, P_F\right) = \left(\begin{array}{c} mP_1 - \frac{N_I}{C}\max\left\{P_F - P_1, 0\right\} \\ mP_F + \frac{\rho N_I}{C}\max\left\{P_F - P_1, 0\right\} - \frac{\rho N_P}{\beta P_0}\max\left\{P_0 - P_F, 0\right\} \end{array}\right)$$

and showing that it is onto. Note that:

$$F\left(P_1, P_F\right) = \left(\begin{array}{c} \mu_1 - \omega_1 \\ E[\tilde{\mu}_2 - \tilde{\omega}_2] \end{array}\right)$$

are precisely the equilibrium conditions.

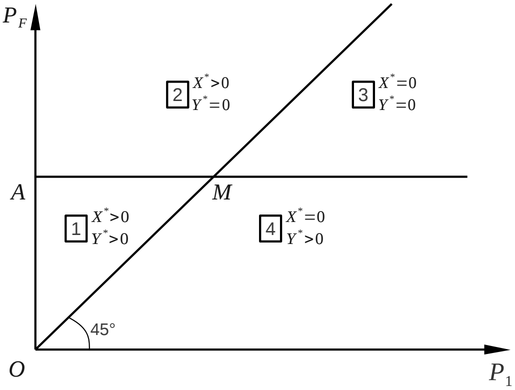


Figure : Phase diagram of physical and financial decisions (exploration).

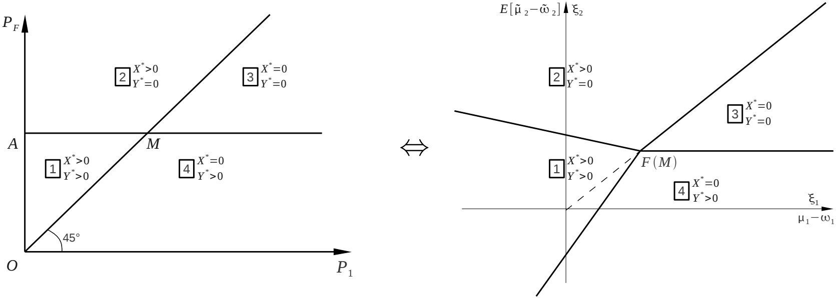


Figure : One-to-one relationship between diagrams.

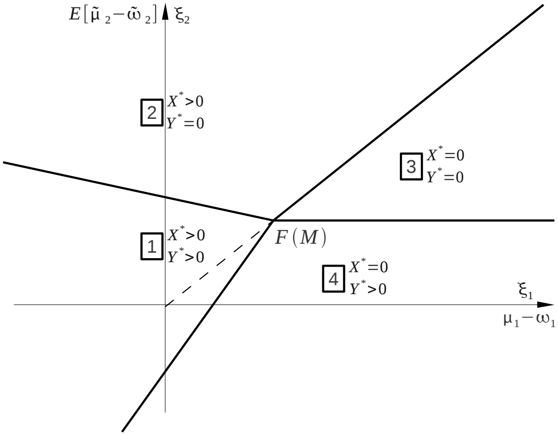


Figure : Phase diagram of physical and financial decisions.

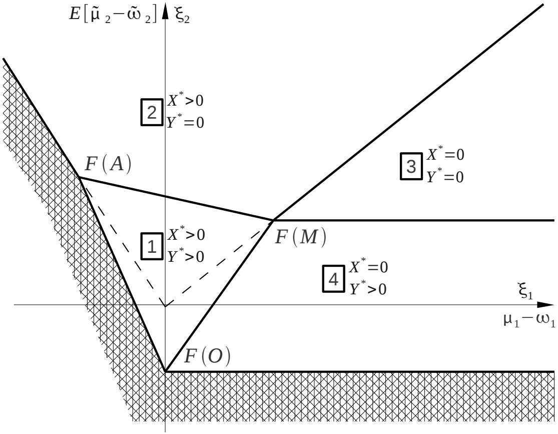


Figure : Phase diagram of physical and financial decisions (final).

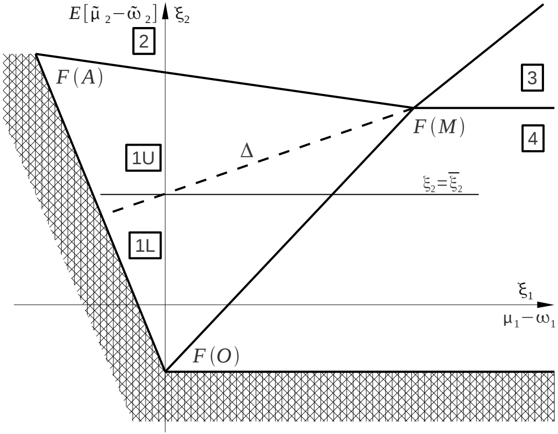
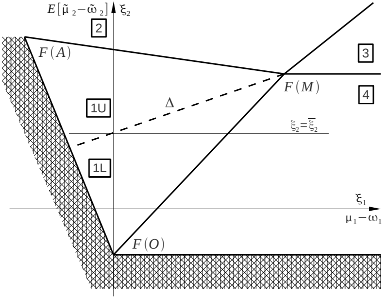


Figure : Zoom on phase diagram of physical and financial decisions (final).



1U	$P_1 < P_F$	$P_F < E[\tilde{P}_2]$	$P_F < P$
	$X^\star > 0$	$f_S > 0$	$Y^\star > 0$
Δ	$P_1 < P_F$	$P_F = E[\tilde{P}_2]$	$P_F < P$
	$X^\star > 0$	$f_S = 0$	$Y^\star > 0$
1L	$P_1 < P_F$	$P_F > E[\tilde{P}_2]$	$P_F < P$
	$X^\star > 0$	$f_S < 0$	$Y^\star > 0$
2	$P_1 < P_F$	$P_F < E[\tilde{P}_2]$	$P_F > P$
	$X^\star > 0$	$f_S > 0$	$Y^\star = 0$
3	$P_1 > P_F$	$P_F = E[\tilde{P}_2]$	$P_F > P$
	$X^\star = 0$	$f_S = 0$	$Y^\star = 0$
4	$P_1 > P_F$	$P_F > E[\tilde{P}_2]$	$P_F < P$
	$X^\star = 0$	$f_S < 0$	$Y^\star > 0$

Figure : Relationships between prices, physical and financial positions.

Political economy and policy evaluation

- Indirect (equilibrium) utilities can be calculated.
- Prices can be calculated.
- Enables various kinds of comparative statics.
- Typical example
  - Impact of # of speculators on the welfare of others.
  - Impact of # of speculators on prices (level, volatility).

Changing the number of speculators

What's that?

- Could be an exogenous, unexpected fact.
    - Metaphor for variations in liquidity,
    - or risk aversion of the rest of the world.
- Could be a decision.
    - Access to derivative markets made easier or stricter.
    - Limits on positions could be changed.
    - Number of licences or any other condition to participate may be changed.
    - Idea that commodity derivatives are for "real" players.

Impact of speculators on prices

Increasing the number of speculators:

- Doesn't influence $\mathrm{E}[\tilde{P}_2 \mid P_1]$ and $\mathrm{Var}[\tilde{P}_2 \mid P_1]$;
- Increases $\mathrm{E}[\tilde{P}_2]$;
- Increases $\mathrm{E}[P_1]$ if $P < \frac{\mathrm{E}[\xi]}{m}$;
- Decreases $\mathrm{E}[P_1]$ if $P > \frac{\mathrm{E}[\xi]}{m}$;
- Increases $\mathrm{Var}[P_1]$ and $\mathrm{Var}[\tilde{P}_2]$.

Speculation may be bad for you

- In our model, the sources of uncertainty (fundamentals of the economy) are exogeneous (ω_1 and ω_2)
- Yet, speculation influences prices !
- In particular, it increases the volatility of prices, and it may even increase their average level
- This is because speculators insure the other participants, thereby encouraging them to take on more risk. The more speculators, the cheaper the insurance premium, the more risk inventory holders and processors are willing to carry.
- Krugman's argument would be correct if horses ("*people buying or selling the stuff*") where not allowed to place bets themselves