

Report on the Hausdorff Trimester Program

Geometry and Physics

May - August 2008

Organizers: Alain Connes, Matthias Lesch, Matilde Marcolli

The program was divided into two main parts. The first, May and June, concentrated on quantum field theories, whereas the second, June and August, focussed around non-commutative geometry. The modern geometric quantum field theory involves mathematical areas like topology, differential geometry and functional analysis, whereas non-commutative geometry has its idea in the fact that a (nice) space is completely characterized by the C^* -algebra of continuous functions, which allows a generalization, mainly propagated by and based on work of A. Connes, by considering non-commutative C^* -algebras. Whereas in the first topic one of the big problems is to construct topological or geometric quantum field theories and to study their properties, a guiding principle for the second topic is to generalize theorems known for manifolds like index theorems to the non-commutative world. In both cases the interaction with theoretical physics is a central theme. Whereas the second topic, non-commutative geometry reflects the interest of the three organizers, the first topic needed additional outside leaders. Fortunately Stephan Stolz and Peter Teichner agreed to play this role.

Both halves of the program organized special activities. Whereas the second part organized workshops, Stolz and Teichner preferred not to do this and instead organized seminars with and lecture series for the participants. In both parts of the program some outstanding visitors took part, besides the organizers and Stolz and Teichner we would like to mention the following names: A. Carey, C. Consani, D. Freed, M. Hopkins, M. Khalkali, H. Moscovici, C. Teleman, W. van Suijlekom.

Here are two papers written in connection with the program, which represent the themes very well. The first is: "The Algebra of Formal Twisted Pseudodifferential Symbols and a Noncommutative Residue" by Farzad Fathi-Zadeh and Masoud Khalkhali. For smooth manifolds (in this context, commutative geometry) residue formulas play a central role. The non-commutative generalization is a difficult task which was achieved by Wodzicki. In the pa-

per a twisted version is defined and some examples are studied. The second paper is: "The diffeomorphism supergroup of a finite-dimensional supermanifold" by Christoph Sachse and Christoph Wockel. In quantum field theory super manifolds play a fundamental role and it is very natural to give the super diffeomorphisms also the structure of a super manifold, of course an infinite dimensional one. This is the subject of this paper.