HIM HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop on

"Geometric Measure Theory and Free Boundary Problems"

February 11-15, 2019

organized by

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Abstracts

Gohar Aleksanyan (University of Duisurg-Essen)

Regularity of the free boundary for the double obstacle problem in dimension two

Abstract: The talk is devoted to the classification of blow-up solutions and to the regularity of the free boundary for the two-dimensional double obstacle problem. In particular, we will see that the double obstacle problem admits a new type of blow-ups, which do not exist in the case of the obstacle problem. We call this new solutions double-cone solutions, since the coincidence set is a union of two cones with a common vertex. We show that if the solution to the double obstacle problem has a double cone blow-up limit, then the blow-up is unique, and locally the free boundary consists of four $C^{1,\alpha}$-curves meeting at a single point. In the end of the talk we will discuss some three-dimensional examples.

Farid Bozorgnia (University of Münster)

On a Class of Singularly Perturbed Elliptic Systems with Asymptotic Phase Segregation

Abstract: This work is devoted to study of a class of elliptic singular perturbed systems and their singular limit to a phase segregating system. We prove existence and uniqueness and study the asymptotic behaviour with convergence to a limiting problem as the interaction rate tends to infinity. The limiting problem is a free boundary problem such that at each point in the domain at least one of the components is zero which implies simultaneously all components cannot coexist. We present a novel method, which provides an explicit solution of limiting problem for special choice of parameters. Moreover, we present some numerical simulations of the asymptotic problem.

Sagun Chanillo (Rutgers University)

Borderline Sobolev Inequalities on Symmetric Spaces with Applications

Abstract: Bourgain and Brezis proved the following remarkable estimate:
Consider the equation,

$$-\Delta u = f$$

where $f : R^n \to R^n$ is a vector field with zero divergence. Further $u : R^n \to R^n$ and the Laplacian acts componentwise. Then one has

$$||\nabla u||_{\frac{n}{n-1}} \leq c(n)||f||_1.$$

We show that the same estimate is valid for non-compact globally symmetric spaces.
Next we show that the original Bourgain-Brezis inequality can be applied to obtain Strichartz inequalities for wave and Schrodinger equations and also we can obtain new estimates for the Maxwell equations and 2D, incompressible Navier-Stokes flow.
These results have been obtained jointly with Jean van Schaftingen and Po-lam Yung.

Alessio Figalli (ETH Zürich)

Generic regularity in obstacle problems

Abstract: The so-called Stefan problem describes the temperature distribution in a homogeneous medium undergoing a phase change, for example ice melting to water. An important goal is to describe the structure of the interface separating the two phases.
In its stationary version, the Stefan problem can be reduced to the classical obstacle problem, which consists in finding the equilibrium position of an elastic membrane whose boundary is held fixed and which is constrained to lie above a given obstacle.
The aim of this talk is to discuss sone recent developments on the generic regularity of the free boundary in both problems.

Jonas Hirsch (University of Leipzig)

Regularity of minimizers for a model of charged droplets

Abstract: We investigate properties of minimisers of a variational model describing the shape of charged liquid droplets. Roughly speaking, the shape of a charged liquid droplet is determined by the competition between an "aggerating" term, due to surface tension forces, and to a "disaggergating" term due to the repulsive effect between charged particles. A classical model for the energy of a droplet, occupying the volume $E \subset \mathbf{R}^3$ and carrying the charge Q is

$$P(E) + \frac{Q^2}{\mathcal{C}(E)};$$

where $P(E)$ stands for the perimeter of E and the repulsive forces is taken into account by

$$\frac{1}{\mathcal{C}(E)} := \inf \left\{ \frac{1}{4\pi} \iint \frac{d\mu(x)d\mu(y)}{|x-y|} : \text{supp } \mu \subset E, \mu(E) = 1 \right\}.$$

A perturbative analysis by Lord Rayleigh showed that the ball is linearly stable for charges Q below a threshold. Experimental it had been confirmed that above a critical value Q_c the appearance of the droplet is changing. One observes the development of singularities, called Taylor's cones, emitting a steady jet of small but highly charged balls.
It had been shown by Goldman, Novaga and Ruffini that the variational problem, that should determine the optimal shape,

$$\min_{|E|=V} P(E) + \frac{Q^2}{\mathcal{C}(E)}.$$

is ill-posed. Muratov and Novaga suggested to consider the following *Deby-Hückel-type free energy* (in any dimension)

$$\mathcal{F}(E, u, \rho) := P(E) + Q^2 \left\{ \int_{\mathbf{R}^n} a_E |\nabla u|^2 \, dx + K \int_E \rho^2 \, dx \right\}.$$

In my talk I want to present our "first" analysis of that second model. In particular we show that minimisers satisfy a partial regularity result, a first step of understanding the further properties of a minimiser.

Yash Jhaveri (Institute for Advanced Study, Princeton)

Higher Regularity of the Singular Set in the Thin Obstacle Problem

Abstract: In this talk, I will give an overview of some of what is known about solutions to the thin obstacle problem, and then move on to a discussion of a higher regularity result on the singular part of the free boundary. This is joint work with Xavier Fernández-Real.

Herbert Koch (University of Bonn)

The thin obstacle problem: Carleman inequalities and higher regularity of the regular part

Abstract: Carleman inequality imply logarithmic convexity of L^2 norms on balls, similar to monotonicity formulas. As consequence blow-ups have a nontrivial limit. The regular part oft the free boundary can be understood as a fully nonlinear subelliptic problem, and hence the regular part of the free boundary is as smooth as the data permits. I will explain these ideas for rough coefficients and the obstacle problem for the fractional Laplacian. The talk is on joint work with Angkana Rüland and Wenhui Shi.

Erik Lindgren (Uppsala University)

Infinity-harmonic potentials in convex rings

Abstract: In this talk, I will discuss certain solutions of the Infinity-Laplace equation in planar convex rings. Focus will be on their streamlines. It turns out that their ascending streamlines are unique while the descending ones may bifurcate. I will explain why bifurcation occurs quite often and the connection to regularity issues. Finally, I will discuss the solution in a punctured square more in detail.

Stephan Luckhaus (University of Leipzig)

Cosserath stractures and small angle grain boundarie for 2d elasticity

Abstract: We show in a joint paper with G.Lauteri available on the archive that small energy dislocation configurations necessarily lead to Cosserath structures.So models involving torsion for plastically deformed metals can only appear either in a nonanealed situation or after a second homogenization.

Edgard Pimentel (PUC-Rio)

Regularity theory for nonlinear PDEs

Abstract: In this talk we discuss recent developments in regularity theory for some classes of nonlinear PDEs. Our arguments relate a problem of interest to another one, for which a richer theory is available.

It operates in two distinct layers; first compactness builds upon suitable notions of stability to produce approximation results. Then, a scaling argument localizes the analysis to establish (in some cases, sharp) regularity results. The toy-models we cover include fully nonlinear PDEs, the Isaacs equation, double-divergence problems and degenerate/singular equations. We close the talk with an excursion into the realm of free boundary problems.

Matthias Röger (TU Dortmund)

Nonlocal perturbations of bending energies

Abstract: We consider a functional on sets in the plane or in three dimensional space, given by contributions from a surface area energy, a bending energy, and a Riesz self-interaction energy. We present results for corresponding minimization problems in classes of fixed volume. (This is joint work with Michael Goldman, Paris, and Matteo Novaga, Pisa.)

Sebastian Schwarzacher (Charles University)

On compressible fluids interacting with a linear-elastic Koiter shell

Abstract: The lecture is about the motion of a viscous incompressible fluid in three dimensions interacting with a flexible shell. The shell constitutes a moving part of the boundary of the physical domain. Its deformation is modeled by a linearized version of Koiter's elastic energy. We discuss the existence of weak solutions to the corresponding system of PDEs. It will be explained that a weak solution exists until the moving boundary approaches a self-intersection. This is a joint work with D. Breit (Heriot-Watt Univ. Edinburgh).

Henrik Shahgholian (KTH)

Free boundaries on Lattice, and their scaling limits

Abstract: Probably the most well-know fact in classical potential theory is the mean value property for harmonic functions over spherical shells or balls. We shall discuss similar properties for harmonic functions on the lattice $s\mathbf{Z}^2$, and show (through numerics) that interesting new objects may appear when the size of lattice s tends to zero. This behaviour hints towards creation of facets in free boundary problems. These objects have been studied in two recent works in collaboration with Hayk Aleksanyan.

Wenhui Shi (University of Heidelberg)

An epiperimetric inequality approach to the parabolic Signorini problem

Abstract: The parabolic Signorini problem concerns solutions to the diffusion equation $\partial_t u - \Delta u = f$ in $\Omega \times (0,T]$ under the Signorini boundary condition

$$u \geq 0,\ \partial_\nu u \geq 0,\ u\partial_\nu u = 0 \text{ on } \partial\Omega \times (0,T], \quad \nu: \text{ outer normal of } \Omega.$$

The solution u satisfies either Dirichlet or Neumann condition on the boundary (depending on whether $u > 0$ or $u = 0$), and the interface between the Dirichlet and Neumann part is the so-called free boundary.

In this talk I will discuss an epiperimetric inequality associated to the parabolic Signornini problem, and show how it can be used to study the asymptotic behavior of the solution around certain free boundary points, as well as the regularity of the free boundary.

Mariana Smit Vega Garcia (Western Washington University)

The fractional unstable obstacle problem

Abstract: We study a model for combustion on a boundary. More specifically, we study certain generalized solutions of a fractional unstable obstacle problem. We study the behavior of the free boundary and prove an upper bound for the Hausdorff dimension of the singular set. We also show that certain symmetric solutions are either stable or unstable depending on the parameter s, for the fractional Laplacian. This is joint work with Mark Allen.

Stephen J. Watson (University of Glasgow)

Emergent Symmetries of the Geometric Ginzburg-Landau Equation

Abstract: The (driven) *Geometric Ginzburg-Landau* equation governs the dynamics of slightly undercooled crystal-melt interfaces [1]. For stongly anisotropic (non-convex) suface energies, we will present the key analytical steps to identify the singular limit of this hyperbolic-parabolic 4^{th}-order geometric partial differential equation. In particular, we will demonstrate the existence of 1D convex- and concave- translating fronts (*solitons*), which are neither mirror images of one another nor possess asymptotic angles that match the thermodynamically expected *Wulff* angles. We will then show how these *thermokinetic* solitons induce a nonlocal emergent facet dynamics via a geometric matched-asymptotic analysis. We will close by touching on how the (emergent) parabolic and Lorentzian symmertry of this facet dynamics, when taken together with the *Principle of G-Equivariant Universality* [2], predict a universal coarsening laws beyond mere scaling [2,3].

References

[1] *Emergent parabolic scaling of nano-faceting crystal growth*
S. J. Watson, Proc. R. Soc. A 471: 20140560 (2015)

[2] *Lorentzian symmetry predicts universality beyond scaling laws*,
SJ Watson, EPL 118 (5), 56001, (Aug.2, 2017) (**Editor's Choice**).

[3] *Scaling Theory and Morphometrics for a Coarsening Multiscale Surface, via a Principle of Maximal Dissipation"*, S. J. Watson and S. A. Norris, Phys. Rev. Lett. 96, 176103 (2006)
