HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop

"Qualitative behaviour of kinetic equations and related problems: numerical and theoretical aspects"

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organized by
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Abstracts

Marianne Bessemoulin-Chatard (CNRS - Université de Nantes)

Hypocoercivity and diffusion limit of a finite volume scheme for linear kinetic equations

Abstract: In this talk, I will present some recent results obtained in collaboration with Maxime Herda (INRIA Lille) and Thomas Rey (Univ. Lille). We are interested in the asymptotic analysis of a finite volume scheme for 1D linear kinetic equations, with either Fokker-Planck or linearized BGK collision operator. On the one hand, we establish that the proposed scheme is Asymptotic Preserving in the diffusive limit. On the other hand, we adapt to the discrete framework the hypocoercivity method proposed by Dolbeault, Mouhot and Schmeiser (2015) to prove the exponential return to equilibrium of the approximate solution, with decay estimates uniform in the diffusive limit.

Emeric Bouin (Université Paris Dauphine)

Hypocoercivity for linear kinetic equations in fractional regimes

Abstract: This talk will report on a work part of the PhD thesis of Laflèche, together with Dolbeault, Mouhot and Schmeiser. I will present how to find decay rates to equilibrium for linear kinetic equations where the microscopic equilibrium is polynomial. Our collision operators may be of three main types : Fokker-Planck, Scattering and Levy-Fokker-Planck. The method is an extension of the Dolbeault-Mouhot-Schmeiser L^2 approach but requires several important changes. Some uniformity in scaling limits will be discussed.

Maxime Breden (Technische Universität München)

On the equilibria of Boltzmann-like collision kernels

Abstract: Collision invariants, i.e. functions $\varphi = \varphi(v)$ such that

$$\left\{ \begin{aligned} v + v_* &= v' + v'_* \\ |v|^2 + |v_*|^2 &= |v'|^2 + |v'_*|^2 \end{aligned} \right\} \implies \varphi(v) + \varphi(v_*) = \varphi(v') + \varphi(v'_*),$$

play a central role in the study of Boltzmann equation for dilute gases. Indeed they characterize the conserved quantities, the possible states for which the entropy dissipation vanish, the kernel of the linearized Boltzmann operator around such equilibria, etc. Boltzmann himself already showed (at least formally) that the only collision invariants are the linear combinations of 1, v_i and $|v|^2$, and this has then later been verified rigorously under very weak regularity assumptions on φ.
In this talk, we address the following question: what are the possible collision invariants, if the conserved energy is no longer the kinetic energy? That is, given a function ω, what are the functions $\varphi = \varphi(v)$ such that

$$\left\{\begin{matrix} v + v_* = v' + v'_* \\ \omega(v) + \omega(v_*) = \omega(v') + \omega(v'_*) \end{matrix}\right\} \implies \varphi(v) + \varphi(v_*) = \varphi(v') + \varphi(v'_*)\,?$$

This question naturally arise in several situations, such as the study of relativistic gases, or the theory of weak turbulence for gravity waves, which can both we described via Blotzmann-like equations where the conserved energy is no longer $|v|^2$. We will show under which assumptions on ω one can generalize Boltzmann's result and prove that 1, v_i and $\omega(v)$ are the only collision invariants.

This is a joint work with Laurent Desvillettes.

Carlos Castro (Polytechnical University of Madrid)

Numerical approximation of the decay rate for some disipative systems

Abstract: It is well known that, under certain hypotheses, the spectrum of a dissipative system characterizes the best decay rate of the solutions. We propose a numerical method to approximate this spectrum when the operator is a bounded perturbation of an unbounded skew-adjoint one. Roughly speaking, it consists in a projection method on finite dimensional subspaces generated by the first eigenfunctions of the unperturbed operator. We illustrate the numerical approach with several experiments for damped wave and beam models. This is a joint work with K. Ammari from University of Monastir (Tunez).

Ludovic Cesbron (Ecole Polytechnique)

A fractional Fick Law method for diffusion limits of kinetic equations

Abstract: In this talk I will present a new method to study the fractional diffusion limits of kinetic equations, recently developed in collaboration with C. Bardos and C. Schmeiser. The basic idea of this method is to generalise the Fick Law which states that the flux of particles through a surface is proportional to the gradient of the density of the fluid through that surface. We will see that when the velocity equilibrium distribution of the kinetic equation is heavy-tailed, i.e. when its second moment is unbounded, the flux of particles will be proportional to a fractional derivative of the density.

Anaïs Crestetto (University of Nantes)

Micro-macro discretizations for collisional kinetic equations of Boltzmann-BGK type in the diffusive scaling

Abstract: This talk aims to present asymptotic preserving (AP) schemes, based on micro-macro decomposition and particle method, for kinetic equations of Boltzmann-BGK type in the diffusive scaling.

The objective is to obtain an efficient numerical method that (i) verifies the AP property, (ii) has a computational cost similar to a fluid scheme when the limit is approached, (iii) is free from too restrictive stability condition, (iv) can be extended and used in three dimensional in space and in velocity (3Dx-3Dv) framework.
A one-order in time discretization will first be presented for the 1Dx-1Dv radiative transport equation. It is based on a suitable formulation of the micro-macro model and the use of weighted particle method for the microscopic (kinetic) part. After that, we will discuss recent extensions: a second-order in time discretization, its application to the 1Dx-1Dv Vlasov-BGK equation, the use of Monte Carlo method (instead of weighted particle method) in the radiative transport equation case and its application to the 2Dx-2Dv and 3Dx-3Dv frameworks.
Numerical results will illustrate the efficiency of this approach.

This work is a collaboration with Nicolas Crouseilles (Univ Rennes and Inria Rennes - Bretagne Atlantique), Giacomo Dimarco (University of Ferrara) and Mohammed Lemou (Univ Rennes and CNRS).

Helge Dietert (Université Paris Diderot)

Trend to equilibrium for a spatially degenerate kinetic Fokker-Planck equation

Abstract: This talk will present some results on the trend to equilibrium problem for the kinetic Fokker-Planck equation in the presence of spatial degeneracies. Our work takes inspiration from the available results in the literature on the hypocoercivity of spatially degenerate linear Boltzmann equation. We show that a certain uniform geometric control condition is only sufficient but not necessary for exponential decay. We demonstrate this by a concrete counterexample.

Megan Griffin-Pickering (University of Cambridge)

A Particle Approximation for the Kinetic Incompressible Euler Equation

Abstract: The Kinetic Incompressible Euler equation is a model for plasma. It is the formal limit of the classical Vlasov-Poisson system in the 'quasineutral' limit where the Debye length tends to zero. The Vlasov-Poisson system can itself be derived formally from a system of interacting particles, in the limit as the number of particles tends to infinity. The rigorous justification of this 'mean field' limit remains a major open problem. However, in recent years, researchers have derived the Vlasov-Poisson equation rigorously from various regularised microscopic systems. In this talk, I will present a joint work with Mikaela Iacobelli, in which we give a rigorous derivation of the Kinetic Incompressible Euler equation from a regularised particle system, using a combined mean field and quasi-neutral limit.

Jessica Guerand (University of Cambridge)

Quantitative regularity for parabolic De Giorgi classes

Abstract: De Giorgi method is a way to prove Hölder regularity of solutions of parabolic equations. While in the elliptic case the proof is completely quantitative, in the parabolic case it seems to remain a non-quantitative step: the intermediate value lemma. The purpose of this talk is to present a quantitative version of this step after introducing how it is useful to get Hölder regularity.

Frédéric Hérau (Nantes University)

A Korn-Wirtinger inequality

Abstract: In kinetic theory or in other fields, some control of the gradient by the symmetric gradient of the macroscopic velocity of a system of particle may be necessary, since only the second quantity appears naturally in physical equations. This type of inequality is known on bounded domains with or without axisymmetries. In this talk, we present a version on the whole space equipped with a probability measure, and give an example where this type of tool may be useful. This is a joint work with K. Carrapatoso, J. Dolbeault, S. Mischler, C. Mouhot, and C. Schmeiser.

Ning Jiang (Wuhan University)

From Vlasov-Maxwell-Boltzmann system to two-fluid incompressible Navier-Stokes-Fourier-Maxwell system with Ohm's law

Abstract: For the two-species Vlasov-Maxwell-Boltzmann (VMB) system with the scaling under which the moments of the fluctuations to the global Maxwellians formally converge to the two-fluid incompressible Navier-Stokes-Fourier-Maxwell (NSFM) system with Ohm's law, we prove the uniform estimates with respect to Knudsen number ε for the fluctuations. As consequences, the existence of the global in time classical solutions of VMB is established. Furthermore, the convergence of the fluctuations of the solutions of VMB to the classical solutions of NSFM with Ohm's law is rigorously justified.
This limit was justified in the recent breakthrough of Arsenio and Saint-Raymond from renormalized solutions of VMB to dissipative solutions of incompressible viscous electro-magneto-hydrodynamics under the corresponding scaling. In this sense, our result gives a classical solution analogue of the corresponding limit in Arsenio and Saint-Raymond's work.

Laurent Laflèche (Université Paris-Dauphine et École polytechnique)

Fractional Hypocoercivity

Abstract: In this talk, we will talk about the large time behaviour of kinetic equations without spatial confinement and with fat tailed local thermodynamical equilibria. It has been proved in most of the cases that such operators can have an anomalous diffusion limit, meaning that in the appropriate scaling, the macroscopic equation is the fractional heat equation.
At the level of the kinetic equation, we develop an L^2 hypocoercivity approach to obtain decay rates towards 0. The method is applied to kinetic equations with various linear collision operators: the Fokker-Planck operator, the Linear Boltzmann operator and the fractional Fokker-Planck operator. The result let appear a comptetition between thermalisation and fractional diffusion.

Donghyun Lee (POSTECH)

Vlasov-Poisson-Boltzmann equation in bounded domains

Abstract: When dilute charged particles are confined in a bounded domain, boundary effects are crucial for dynamics of particles which can be modeled by the Vlasov-Poisson-Boltzmann system. Considering a diffuse boundary condition, we construct a unique global-in-time solution in convex domains. This construction is based on $L^2 - L^\infty$ framework with a new weighted $W^{1,p}$-estimate of distribution functions and $C^{2,\alpha}$-estimate of self-consistent electric potentials. Furthermore we prove an exponential convergence of distribution functions toward a global Maxwellian.

Vivion Léo (Université Cote d'Azur)

Landau damping in dynamical Lorentz gases

Abstract: In 2001 L. Bruneau and S. De Bièvre introduced an original model describing the interaction between a classical particle and an oscillating media. They showed (under some conditions) that for this model the media acts on the particle like a friction force. Here, I will present results about the Vlasov-Wave equation: the kinetic equation coming from their model.
Since in 2016 S. De Bièvre, T. Goudon and A. Vavasseur showed that, under an appropriate scaling, the Vlasov-Wave equation is asymptotic to the attractive Vlasov-Poisson equation, it is natural to ask the question of the Landau damping for this model. First, I will recall the mechanisms of the Landau damping for the classical Vlasov equation and then I will explain how these mechanisms are modified in the Vlasov-Wave case. All these results will be illustrated by numerical simulations.

Pierre Monmarché (Sorbonne Université)

Bouncing with velocity jump process

Abstract: A class of velocity jump processes have emerged from stochastic algorithm motivations. The motion of a particle is similar to the case of BGK equations or to Run-and-Tumble processes that model motion of self-propelled bacteria, i.e. the velocity of the particle is piecewise constant with random jumps. In the Markov Chain Monte Carlo algorithm case, the definition of the collision rate and kernel is not motivated by physical considerations, but ensures that the equilibrium of the process is some prescribed target measure. I will give a general introduction of these processes and present a few results about metastability or ergodicity.

Arnaud Münch (Clermont-Auvergne University)

Asymptotic and null controllability of an advection-diffusion equation

Abstract: We discuss the asymptotic analysis of the solution of the advection-diffusion equation $y_t - \varepsilon y_{xx} + y_x = 0$ with respect to the parameter $\varepsilon > 0$. We describe notably the interaction of the boundary and internal layers. We also discuss the null controllability property of the equation as ε goes to zero.

Karel Pravda-Starov (Université de Rennes 1, France)

Sufficient geometric conditions for the null-controllability of evolution equations enjoying Gelfand-Shilov smoothing properties

Abstract: We consider evolution equations enjoying Gelfand-Shilov smoothing properties in any positive time and study how the indices of Gelfand-Shilov regularity relate to the geometry needed on the control subset to ensure the null-controllability in any positive time of these evolution equations. To that end, we establish new uncertainty principles for finite combinations of Hermite functions with an explicit control of the constant with respect to the energy level of the Hermite functions as eigenvalues of the harmonic oscillator. Some applications to the null-controllability of the evolution equations associated to fractional harmonic oscillators or certain classes of hypoelliptic non-selfadjoint quadratic equations are given. This is a joint work with Jérémy Martin.

L. Miguel Rodrigues (Univ Rennes & Institut Universitaire de France)

Large-magnetic field regimes and asymptotic preserving schemes

Abstract: Even in highly-oscillating dynamics may persist quantities that are evolving on slower scales and, at first-order, uncouple from fast oscillations. Thus asymptotically the slow dynamics obeys a closed system of uncoupled equations, which may be thought as averaged equations.

Francesco Salvarani (Università di Pavia)

On the homogenization problem for the linear Boltzmann equation

Abstract: In this talk, we study the homogenization problem for the linear Boltzmann equation when the optical parameters are highly heterogeneous in the energy variable. We employ the method of two-scale convergence to arrive at the homogenization result. In doing so, we show the induction of a memory effect in the homogenization limit. The results presented here have been obtained in collaboration with Harsha Hutridurga and Olga Mula.

Havva Yoldaş (BCAM / University of Granada)

Harris's Theorem and its applications to some kinetic and biological models for long-time behaviour of solutions

Abstract: In this talk, we give a brief explanation of Harris's Theorem and its precursor Doeblin's Theorem which are developed for the study of discrete-time Markov chains. This probabilistic approach is based on quantitatively verifying a minorisation condition and a drift condition in order to obtain quantitative estimates on the long-time behaviour of time evolution for some linear and non-local integro-PDEs. We will see how this method applies to some models in population dynamics and kinetic theory.
