HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop on

"Workshop on Harmonic analysis, Singular Integrals and PDEs"

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organized by
Marianna Csörnyei (Chicago), Tuomas Orponen (Jyväskylä), Xavier Tolsa (Barcelona), Tatiana Toro (Washington), Alexander Volberg (Michigan)

Abstracts

Tomasz Adamowicz (IMPAN Warsaw, Poland)

Geometry of level sets of harmonic functions and p-harmonic mappings: convexity, curvature, isoperimetric inequalities for PDE and three-spheres theorems

Abstract: One of the main themes of the talk are monotonicity formulas for level sets of harmonic functions in Euclidean domains and Riemannian surfaces, including the singular Alexandrov surfaces. Such formulas allow for studying the logarithmic convexity of the length of the level curves and related isoperimetric type inequalities. Related are the studies of the geodesic curvature of the level curves and of the steepest descent. Moreover, we present the arithmetic three-spheres theorem for coordinate functions of p-harmonic mappings and discuss its relation to the unique continuation problem. The talk is partially based on the joint work with Giona Veronelli.

Dmitriy Bilyk (University of Minnesota, USA)

Discrete minimizers of energy integrals

Abstract: It is quite natural to expect that minimization of pairwise interaction energies leads to uniform distributions, at least for "nice" kernels. However, the opposite effect occurs in many interesting examples, especially for attractive-repulsive energies or when the repulsion is very weak: minimizing measures are discrete (or at least are very non-uniform, e.g. supported on "thin" or lower-dimensional sets). We shall discuss some results related to this curious phenomenon and its relation to analysis, signal processing, discrete geometry etc.

Chandan Biswas (Indian Institute of Science, Bangalore)

On extremizers for Fourier restriction onto the moment curve

Abstract: We will discuss the existence of a function which saturates the operator norm of the Fourier restriction/extension operator corresponding to the moment curve $(t, t^2, \dots, t^d)$ in Lebesgue spaces. This is based on our recent work with Betsy Stovall.

Paolo Bonicatto (University of Warwick, UK)

Moving currents: on the Lie transport equation and a Rademacher type theorem

Abstract: In the classical theory, given a vector field $b\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$, one usually studies the transport/continuity equation drifted by b looking for solutions in the class of functions (with certain integrability) or at most in the class of measures. In this seminar I will talk about recent efforts, motivated by the modeling of defects in plastic materials, aimed at extending the previous theory to the case when the unknown are instead k-currents in $\mathbb{R}^d$, i.e. generalised k-dimensional surfaces. The resulting equation involves the Lie derivative L_b of currents in direction b and reads $\partial_t T_t + L_b T_t = 0$. It is easily seen that the continuity equation corresponds to the case of 0-currents, while the transport equation to the case of d-currents. I will explain the main challenges this problem presents and some recent results based on an ongoing project with Giacomo Del Nin and Filip Rindler (University of Warwick).

Mingming Cao (Instituto de Ciencias Matemáticas, Spain)

Absolute continuity of elliptic measure in 1-sided NTA domains satisfying CDC

Abstract: Let $\Omega \subset \mathbb{R}^{n+1}$, $n \geq 2$, be a 1-sided NTA domain satisfying CDC. Let $L_0 u = -\mathrm{div}(A_0 \nabla u)$, $Lu = -\mathrm{div}(A\nabla u)$ be two real uniformly elliptic operators in Ω, and write ω_{L_0}, ω_L for the respective associated elliptic measures. We establish the equivalence between the following: (i) $\omega_L \in A_\infty(\omega_{L_0})$, (ii) L is $L^p(\omega_{L_0})$-solvable, (iii) bounded null solutions of L satisfy Carleson measure estimates with respect to ω_{L_0}, (iv) $\mathcal{S} < \mathcal{N}$ (i.e., the conical square function is controlled by the non-tangential maximal function) in $L^q(\omega_{L_0})$ for any null solution of L, and (v) L is $\mathrm{BMO}(\omega_{L_0})$-solvable.

Alan Chang (Princeton University, USA)

Analytic capacity and projections

Abstract: Given a planar set E and a Borel measure μ supported on E, we study the connection between the analytic capacity of E and the L^2 norm of the orthogonal projections of μ. This is related to an old conjecture of Vitushkin about the relationship between the Favard length and analytic capacity. This is joint work with Xavier Tolsa.

Damian Dąbrowski (University of Jyväskylä, Finland)

Vitushkin's conjecture and sets with plenty of big projections

Abstract: In this talk I am going to describe recent progress made on Vitushkin's conjecture: if a compact set has plenty of big projections, then it is non-removable for bounded analytic functions. Based on joint work with Michele Villa.

Stefano Decio (NTNU, Norway)

Bounds on the Hausdorff measure of zero sets of Steklov eigenfunctions

Abstract: Steklov eigenfunctions in a bounded domain are harmonic functions whose normal derivative at the boundary is proportional to the function itself. I will discuss recent results on the Hausdorff measure of their zero sets, focusing mainly on upper bounds. Comparisons with the somewhat better understood case of Laplace eigenfunctions will be provided.

Jaume de Dios Pont (University of California, Los Angeles)

Decoupling for Cantor sets and the parabola

Abstract: Decoupling estimates aim to study the "amount of cancellation" that can occur when we add up functions whose Fourier transforms are supported in different regions of space. In this talk I will describe decoupling estimates for a Cantor set supported in the parabola. I will discuss how both curvature and sparsity (or lack of arithmetic structure) can separately give rise to decoupling estimates, and how these two sources of "cancellation" can be combined to obtain improved estimates for sets that have both sparsity and curvature. No knowledge of what a decoupling estimate is will be assumed.
Based on joint work with Alan Chang, Rachel Greenfeld, Asgar Jamneshan, José Madrid and Zane Li.

Giacomo Del Nin (University of Warwick, UK)

Endpoint Fourier restriction and unrectifiability

Abstract: In this talk we show that if a measure of dimension s on $\mathbb{R}^d$ admits (p, q) Fourier restriction for some endpoint exponents allowed by its dimension, namely $q = \frac{s}{d}p'$ for some $p > 1$, then a dichotomy holds: the measure is either absolutely continuous or 1-purely unrectifiable. Based on a joint work with Andrea Merlo (Paris-Saclay).

Max Engelstein (University of Minnesota, Twin Cities)

Harmonic analysis techniques for (Almost-)Minimizers

Abstract: Tools from free boundary theory and the calculus of variations have been critical in studying the relationship between the behavior of harmonic measure and the geometry of domains. In this talk we "give back" and describe some ways in which harmonic analysis and geometric measure theory inform the study of (almost-)minimizers to the one and two-phase Alt-Caffarelli functionals. This is based on joint work (and some upcoming work) with Guy David, Mariana Smit Vega Garcia and Tatiana Toro.

Joseph Feneuil (Université Paris-Saclay, France)

Carleson estimates on solutions in domains with uniformly rectifiable boundaries

Abstract: Let $\Omega \subset \mathbb{R}^n$ be a domain with a d-dimensional uniformly rectifiable boundary. If $d < n-1$, it means that Ω is the complement of its boundary, and if $d = n-1$, we also assume sufficient connectedness inside the domain (1-sided NTA). We look at elliptic operators in the form $L = -\mathrm{div} A\nabla$,

where the coefficients A satisfy some regularity (Dahlberg-Kenig-Pipher operators). We will present estimates on the elliptic measure (elliptic measure is A_∞ with respect to the Hausdorff measure) and the Green function (the Green function is close to a distance) associated to L.

Max Goering (Max Planck Institute, Germany)

Regularity and a degenerate class of PDEs stemming from anisotropic minimal surfaces

Abstract: In the 1950s, De Giorgi introduced the now standard "tilt-excess decay" argument, to verify the regularity of area minimizing surfaces in a neighborhood of flat points. A key step of De Giorgi's argument was the regularity of the minimal surface equation when u is a Lipschitz function–the motivation for his contributions to De Giorgi-Nash-Moser theory. By analyzing surfaces which locally minimize the energy $\Phi_p(E;A) = \int_{\partial^* E \cap A} \|\nu_E\|_p d\mathcal{H}^{n-1}$, we motivate studying the broad class of PDEs which take the form div $\left(\rho(Du)^{\gamma-1}(D\rho)(Du)\right) = 0$ for some norm ρ and some $\gamma > 1$. These PDEs often correspond to the "small-gradient approximation" of anisotropic minimal surface equations. Early results toward the higher-order regularity of solutions to this class of PDEs will be presented.

John Hoffman (University of Missouri, USA)

Parabolic Quantitative Rectifiability and Singular Integral Operators

Abstract: In this talk, we discuss a recent result which says that parabolic Ahlfors-David regular sets on which sufficiently "nice" singular integral operators are bounded from L^2 to L^2 are exactly those which are Parabolic Uniformly Rectifiable. This is an analog of a result proved by David and Semmes in 1991 in the Elliptic setting. Our proof relies on the "Corona Decomposition" – a powerful tool that can be used to link the analytic and geometric characterizations of Uniform Rectifiability. We will highlight some of the significant differences between the Elliptic and Parabolic theories of Uniform Rectifiability and show how these appear in our proof. This talk is based on joint work with Bortz, Hofmann, García, and Nyström.

Steve Hofmann (University of Missouri, USA)

Quantitative rectifiability in the parabolic setting: a survey of recent progress

Abstract: In the 1990's, David and Semmes developed a remarkable theory of quantitative rectifiability, which in particular characterized geometrically the sets on which singular integrals of Calderón-Zygmund type are bounded on L^2. In more recent years this theory has played a central role in the geometric characterization of those domains in which the Dirichlet problem is solvable with boundary data in L^p, for some $p < \infty$, thus providing an analogue of the Wiener criterion, but for singular data rather than continuous data. In this talk we discuss recent work taking initial steps to extend the David-Semmes theory to the parabolic setting. This is joint work with Simon Bortz, John Hoffman, Chema Martell, and Kaj Nyström.

Benjamin Jaye (Georgia Tech, USA)

Two Extremal Classes of Measures Associated to Singular Integral Operators

Abstract: In this talk we will discuss two classes of measures associated to a singular integral operator: reflectionless measures; and symmetric measures. We will attempt to describe how these classes of

measures arise naturally in problems concerning the geometry of measures for which a singular integral operator is well behaved, and discuss what is known (and unknown) about these two classes of measures in terms of available techniques and results.

Alexander Logunov (Université de Genève, Switzerland)

An elliptic adaptation of ideas of Carleman and Domar from complex analysis related to Levinson's log log theorem

Abstract: Using the three balls inequality, we adapt the elegant ideas of Carleman and Domar from complex analysis to linear elliptic PDE and generalize the classical Levinson's loglog theorem.

José María Martell (ICMAT CSIC-UAM-UC3M-UCM, Spain)

Layer potentials, Extrapolation, and Boundary Value Problems in unbounded domains

Abstract: In joint recent work with J.J. Marín, D. Mitrea, I. Mitrea, and M. Mitrea we study BVP for elliptic systems with constant complex coefficients in unbounded domains whose unit normal has sufficiently small oscillation. Using the method of layer potentials we construct solutions, which are indeed unique, after showing that some natural operator is invertible by using a Neumann series. Our methods allow us to consider BVP with boundary data in Lebesgue spaces with Muckenhoupt weights. This, together with some sharpened version of the Rubio extrapolation theorem, give well-posdeness of BVP in weighted Banach function spaces.

Mihalis Mourgoglou (Universidad del País Vasco and Ikerbasque, Spain)

The regularity problem for the Laplace equation in rough domains

Abstract: In this talk I will present some recent advances on Boundary Value Problems for the Laplace operator with rough boundary data in a bounded corkscrew domain in $\mathbb{R}^{n+1}$ whose boundary is uniformly n-rectifiable and its measure theoretic boundary agrees with its topological boundary up to a set of n-dimensional Hausdorff measure zero. In particular, I will discuss the equivalence between solvability of the Dirichlet problem for the Laplacian with boundary data in $L^{p'}$ and solvability of the regularity problem for the Laplacian with boundary data in an appropriate Sobolev space $W^{1,p}$, where $p \in (1, 2+\varepsilon)$ and $1/p + 1/p' = 1$. As two-sided chord-arc domains satisfy the aforementioned geometric assumptions, our result answers a question posed by Carlos Kenig in 1991. This is joint work with Xavier Tolsa.

Kaj Nyström (Uppsala University, Sweden)

Parabolic operators: fractional powers, weights and Kato

Abstract: In this talk I will discuss some recent results concerning second order parabolic operators with complex coefficients and fractional powers thereof. This leads us to study weighted equations and the Kato square root problem for weighted parabolic operators.

Bruno Poggi (Universitat Autònoma de Barcelona)

Some eigenvalue counting problems for the magnetic Schrödinger operator and their solutions via the Filoche-Mayboroda landscape function

Abstract: In two papers in the 90's, Z. Shen studied non-asymptotic bounds for the eigenvalue counting function of the magnetic Schrödinger operator in a few settings. But in dimensions 3 or above, his methods required a strong scale-invariant assumption on the gradient of the magnetic field; in particular, this excludes many singular or irregular magnetic fields, and the questions of treating these later cases had remained open.
In this talk, we present our solutions to these questions, and other new results on the exponential decay of solutions (eigenfunctions, integral kernels, resolvents) to Schrödinger operators. We will introduce the Filoche-Mayboroda landcape function for the (non-magnetic) Schrödinger operator, present its connection to the classical Fefferman-Phong-Shen maximal function, and then show how one may use directionality assumptions on the magnetic field to construct a new landscape function in the magnetic case. We solve Shen's problems (and recover other results in the irregular setting) by putting these observations together.

Martí Prats (Universitat de Barcelona)

The two-phase problem for harmonic measure in VMO via jump formulas for the Riesz transform

Abstract: Let $\Omega^+ \subset \mathbb{R}^{n+1}$ be an NTA domain and let $\Omega^- = \mathbb{R}^{n+1} \setminus \overline{\Omega^+}$ be an NTA domain as well. Denote by ω^+ and ω^- their respective harmonic measures. Assume that Ω^+ is a δ-Reifenberg flat domain for some $\delta > 0$ small enough. In a joint work with X. Tolsa we show that $\log \frac{d\omega^-}{d\omega^+} \in \mathrm{VMO}(\omega^+)$ if and only if Ω^+ is vanishing Reifenberg flat, Ω^+ and Ω^- have joint big pieces of chord-arc subdomains, and the inner unit normal of Ω^+ has vanishing oscillation with respect to the approximate normal.
This result can be considered as a two-phase counterpart of a more well known related one-phase problem for harmonic measure solved by Kenig and Toro.

Carmelo Puliatti (Universidad del País Vasco, Spain)

L^2-boundedness of gradients of single layer potentials for elliptic operators with coefficients of Dini mean oscillation-type

Abstract: We consider a uniformly elliptic operator L_A in divergence form associated with an $(n+1)\times(n+1)$-matrix A with real, bounded, and possibly non-symmetric coefficients. If a proper L^1-mean oscillation of the coefficients of A satisfies suitable Dini-type assumptions, we prove the following: if μ is a compactly supported Radon measure in $\mathbb{R}^{n+1}$, $n \geq 2$, and $T_\mu f(x) = \int \nabla_x \Gamma_A(x,y) f(y)\, d\mu(y)$ denotes the gradient of the single layer potential associated with L_A, then

$$1 + \|T_\mu\|_{L^2(\mu)\to L^2(\mu)} \approx 1 + \|\mathcal{R}_\mu\|_{L^2(\mu)\to L^2(\mu)},$$

where $\mathcal{R}_\mu$ indicates the n-dimensional Riesz transform. This makes possible to obtain direct generalization of some deep geometric results, initially obtained for $\mathcal{R}_\mu$, which were recently extended to T_μ under a Hölder continuity assumption on the coefficients of the matrix A.
This is a joint work with Alejandro Molero, Mihalis Mourgoglou, and Xavier Tolsa.

Keith Rogers (Instituto de Ciencias Matemáticas, Spain)

Improved bounds for the Kakeya conjecture using semialgebraic geometry

Abstract: The Kakeya problem considers tubes that point in different directions and the degree to which they can be compressed by positioning them strategically. On the one hand, we will see that the measure of any semialgebraic set that contains the tubes must satisfy the expected lower bound. The proof employs tools from real algebraic geometry including Gromov's algebraic lemma and the Tarski-Seidenberg projection theorem. On the other hand, the expected bound holds in the absence of algebraic structure, by polynomial partitioning. Balancing between the two extremes yields improved bounds for the Kakeya maximal conjecture in higher dimensions. This is joint work with J. Hickman, N. Katz and R. Zhang.

Mariana Smit Vega Garcia (Western Washington University, USA)

Almost minimizers for obstacle problems

Abstract: In the applied sciences one is often confronted with free boundaries, which arise when the solution to a problem consists of a pair: a function u (often satisfying a PDE), and a set where this function has specific behavior. Two central issues in the study of free boundary problems are: (1) What is the optimal regularity of the solution u? (2) How smooth is the free boundary? The study of the classical obstacle problem - one of the most renowned free boundary problems - began in the '60s with the pioneering works of G. Stampacchia, H. Lewy, and J. L. Lions. In contrast to the classical obstacle problem, which arises from a minimization problem (as many other PDEs do), minimizing problems with noise leads to the notion of almost minimizers. In this talk, I will introduce obstacle-type problems and overview recent developments in almost minimizers for the thin obstacle problem, illustrating techniques that can be used to tackle questions (1) and (2) in various settings.

Krystal Taylor (The Ohio State University, USA)

Finite point configurations and Newhouse thickness

Abstract: A vibrant and classic area of research is that of relating the size of a set to the finite point configurations that it contains. In the fractal setting, size may refer to dimension or measure. In this talk, we will consider two notions of size- Hausdorff dimension and Newhouse thickness- that can be used to guarantee the existence of arbitrarily long paths within fractal subsets of Euclidean space.

Sergey Tikhonov (Centre de Recerca Matemàtica and ICREA, Spain)

Hardy-Littlewood inequalities for Fourier transforms

Abstract: We discuss classical Hardy-Littlewood-Paley inequalities for Fourier coefficients/transforms as well as their possible extensions for any $1 < p < \infty$.

Michele Villa (University of Oulu, Finland)

Quantitative affine approximation on uniformly rectifiable sets

Abstract: A basic fact of Lipschitz functions is that they are differentiable almost everywhere. This is Rademacher's theorem. It says a lot about the asymptotic behaviour of a Lipschitz function f

at small scales (where they look affine) but not much at any definite scale. How long do we need to wait for the the smoothness of f to kick in and make it look like an affine map? Dorronsoro's theorem is a quantification of Rademacher's which tells us: not long - and indeed, a Lipschitz function looks approximately affine at most scales (in some precise sense). Rademacher's theorem holds for functions on rectifiable sets by a theorem of Federer. In this talk, I will discuss (quantitative) affine approximation results for Lipschitz and Sobolev functions on (quantitatively) rectifiable sets. Based on a joint work with Jonas Azzam and Mihalis Mourgoglou.

Hong Wang (University of California, Los Angeles, USA)

Distance sets spanned by sets of dimension $d/2$

Abstract: Suppose that E is a subset of $\mathbb{R}^d$, its distance set is defined as $\Delta(E) = \{|x-y| : x, y \in E\}$. Joint with Pablo Shmerkin, we prove that if the packing dimension and Hausdorff dimension of E both equal $d/2$, then $\dim_{\mathrm{H}} \Delta(E) = 1$. We also prove that if $\dim_{\mathrm{H}} E \geq d/2$, then $\dim_{\mathrm{H}} \Delta(E) \geq d/2 + c_d$ when $d = 2, 3$; and $\underline{\dim}_{\mathrm{B}} \Delta(E) \geq d/2 + c_d$ when $d > 3$ for some explicit constants $c_d > 0$.
