HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Follow-Up-Workshop to TP

"Harmonic Analysis and Partial Differential Equations"

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organized by
Herbert Koch, Daniel Tataru, Christoph Thiele

Abstracts

Stefan Buschenhenke (Christian-Albrachts-Universität zu Kiel)

The non-linear Brascamp-Lieb-inequality

Abstract: We prove a nonlinear variant of the general BrascampLieb inequality. In-stances of this inequality are quite prevalent in analysis, and we illustrate this with sub-stantial applications in harmonic analysis and partial differential equations. Our proof consists of running an efficient, or tight, induction on scales argument, which uses the existence of gaussian near-extremisers to the underlying linear BrascampLieb inequality (Liebs theorem) in a fundamental way. A key ingredient is an effective version of Liebs theorem, which we establish via a careful analysis of near-minimisers of weighted sums of exponential functions.

Emanuel Carneiro (ICTP - Trieste)

Sobolev regularity for maximal operators

Abstract: I will discuss some of the recent developments on the regularity theory for maximal operators, when acting on Sobolev or BV data. This is an active topic of research with several interesting open questions. The talk should be accessible to a broad audience.

Michael Christ (University of California, Berkeley)

Multilinear oscillatory integral inequalities: Best of the best

Abstract: Consider $T_\lambda^\phi(f_1,\ldots,f_d) = \int_{[0,1]^d} e^{i\lambda\phi(x)} \prod_{j=1}^d f_j(x_j)\,dx$ with ϕ a C^ω real-valued phase function. What is the largest exponent γ for which an upper bound $O(\lambda^{-\gamma}\prod_j \|f_j\|_\infty)$ holds, uniformly for all functions f_j and large positive parameters λ? There is a large and successful body of work for $d = 2$. For the multilinear case, optimal exponents have been determined by Phong-Stein-Sturm, by Gilula-Gressman-Xiao, and by others, with L^{p_j} norms on the right-hand side, for certain ranges of exponents under natural nondegeneracy hypotheses on ϕ. The largest optimal exponent arises for L^∞ norms, which typically lie outside the parameter ranges of earlier works. Exponents γ obtained in those works do not exceed $\frac{1}{2}$.

Examples demonstrate that certain basic themes of earlier works are violated in the L^∞ (or L^p for large p) regime. The main result breaks the barrier $\gamma = 1/2$ in substantial generality for $d = 3$, without yielding optimal exponents. Themes include dichotomy between structure and pseudorandomness, notions of degeneracy of phases, multi-scale analysis, and a connection with certain sublevel sets. A variant of the familiar notion of a sublevel set arises.

Benjamin Dodson (Johns Hopkins University)

Global well-posedness and scattering for the defocusing, cubic wave equation

Abstract: In this talk we discuss a recent result, proving global well-posedness and scattering for the cubic wave equation in three dimensions with radial data. The initial data lies in $\dot{H}^{1/2}(\mathbf{R}^3) \times \dot{H}^{-1/2}(\mathbf{R}^3)$, the critical Sobolev space.

Roland Donninger (University of Vienna)

Strichartz estimates for the one-dimensional wave equation

Abstract: I will report on work in progress with Irfan Glogic on one-dimensional wave evolution in hyperboloidal coordinates. We prove a set of Strichartz estimates for equations perturbed by a general potential. I will also outline possible applications, e.g. Yang-Mills fields on wormhole spacetimes.

Shaoming Guo (University of Wisconsin Madison)

Decoupling inequalities for moment manifolds associated to Arkhipov-Chubarikov-Karatsuba systems

Abstract: I will present decoupling inequalities for a class of moment manifolds. These inequalities imply optimal mean value estimates for multidimensional Weyl sums of the kind considered by Arkhipov, Chubarikov, and Karatsuba and by Parsell. Joint work with Pavel Zorin-Kranich.

Sebastian Herr (Bielefeld University)

Wave Maps and Bilinear Fourier Restriction

Abstract: We will show that an atomic version of the bilinear Fourier restriction theorem for the cone can be used to solve the division problem for the wave maps equation and to provide a new proof of small data global well-posedness and scattering in the critical Besov space. This is joint work with Timothy Candy.

Dirk Hundertmark (KIT)

Counting bound states in quantum mechanics with maximal Fourier–multiplier estimates

Abstract: There are a couple of proofs by now for the famous Cwikel–Lieb–Rozenblum (CLR) bound, which is a bound on the number of bound states for a Schrödinger operator with the correct semiclassical behavior, proven in the 1970s. Of the rather distinct proofs by Cwikel, Lieb, and Rozenblum, the one by Lieb gives the best constant, the one by Rozenblum does not seem to yield any reasonable

estimate for the constants, and Cwikel's proof is said to give a constant which is at least about 2 orders of magnitude off the truth. This situation did not change much during the last 40+ years.
It turns out that this common belief, i.e, Cwikel's approach yields bad constants, is not set in stone: Using bounds for maximal Fourier–multipliers we give a drastic simplification of Cwikel's original approach which leads to an astonishingly good bound for the constant in the CLR inequality. Our proof is also quite flexible and leads to rather precise bounds for a large class of Schrödinger-type operators with generalized kinetic energies. Moreover, it highlights a natural but overlooked connection of the CLR bound with bounds for maximal Fourier–multipliers from harmonic analysis.

Tuomas Hytönen (University of Helsinki)

Dyadic representation meets operator-valued kernels

Abstract: The dyadic representation theory of singular integrals arose from the work on the A_2 conjecture on sharp weighted inequalities. In this original application, it has now been largely surpassed by the sparse domination technology. I will discuss some other types of applications in understanding more complicated bi-parameter singular integrals as formally simpler one-parameter singular integrals but with an operator-valued (in fact, singular integral -valued) kernel. While this basic idea goes back to the work of Journé in the 1980's, it turns out to combine very nicely with the dyadic representation theory. While the dyadic representation of usual singular integrals expresses them as a series of so-called dyadic shifts and paraproducts, the bi-parameter theory features new hybrid combinations that can be conveniently interpreted as paraproduct-valued dyadic shifts. (The talk reports on joint work with Henri Martikainen and Emil Vuorinen.)

Mihaela Ifrim (University of Wisconsin Madison)

Dispersive decay of small data solutions for the KdV equation

Abstract: We consider the Korteweg-de Vries (KdV) equation, and prove that small localised data yields solutions which have dispersive decay on a quartic time-scale. This result is optimal, in view of the emergence of solitons at quartic time, as predicted by inverse scattering theory.

Paata Ivanishvili (University of California, Irvine)

Weissler's conjecture on the boolean cube

Abstract: Let $1 \leq p \leq q$ and z be a complex number. The necessary and sufficient condition for L^p to L^q boundedness of the Hermite semigroup $e^{z\Delta}$ on the boolean cube of an arbitrary dimension equipped with uniform counting measure will be shown to be $|p-2-e^{2z}(q-2)| \leq p-|e^{2z}|q$. This solves an old open problem in complex hypercontractivity theory on the Hamming cube. Certain cases of the triples (p, q, z) were characterized by Bonami (1970); Beckner (1975); and Weissler (1979). Several applications will be presented. Work in progress with Fedja Nazarov.

Rowan Killip (UCLA)

KdV on H^{-1} and beyond

Abstract: I will outline the proof of wellposedness of KdV on $H^{-1}(R)$. To the extent time allows, I will then describe some subsequent developments.

Vjekoslav Kovač (University of Zagreb)

On maximal and variational Fourier restriction

Abstract: The problem of restricting the Fourier transform of a non-integrable function to a hypersurface gained attention in the 1970s and is an active research field today. On the other hand, maximal Fourier restriction estimates were inaugurated recently by Müller, Ricci, and Wright in order to give a pointwise meaning to the restriction of the Fourier transform of a function. We will discuss variational Fourier restriction estimates, which quantify the pointwise convergence in question. Moreover, we will sketch a simple abstract argument that deduces certain maximal and variational Fourier restriction estimates from ordinary a priori estimates. This is joint work with Diogo Oliveira e Silva.

Joachim Krieger (EPFL)

Randomization improved Strichartz estimates and applications

Abstract: It is well-known that the range of available Strichartz estimates for the free wave equation on $n+1$, $n \geq 2$,is improved if one restricts to radial data, particularly in low dimensions. In this talk, we explain how a suitable randomisation procedure allows to obtain similar estimates for non-radial randomised data. This can be used to prove global well-posedness in supercritical data regimes for certain wave equations of wave-maps type. The talk is based on joint work with Nicolas Burq.

Jeremy L. Marzuola (University of North Carolina at Chapel Hill)

Nodal deficiency, spectral flow, and the Dirichlet-to-Neumann map

Abstract: We discuss the Maslov index in the study of eigenvalues and eigenfunctions for second order elliptic operators that arose in recent work with Greg Berkolaiko and Graham Cox. We will also discuss some newer results in the areas of nodal domains with Tom Beck and Yaiza Canzani.

Paul F. X. Müller (J. Kepler University, Linz)

Best Constants in Complex Convexity Estimates, and Extensions to R^n, $n \geq 2$

Abstract: In this talk we report on joint work with A. Lindenberger and M. Schmuckenschläger (JKU Linz). Let σ denote the normalized Haar measure on the sphere S^{n-1}. We show that for all $x \in R^n$, $a \in R^+$ and $p \in (0,1]$:

$$\Big(\int_{S^{n-1}} \|x - az\|^p \,\sigma(dz)\Big)^{1/p} \geq \Big(\|x\|^2 + \frac{p+n-2}{n}\, a^2\Big)^{1/2} .$$

Moreover $C(n,p) = (n+p-2)/n$ is the best possible constant. The cases $n=2$, $n=3$ and $n \geq 4$ are treated by separate methods.
In the special case $n=2$ this gives the best constant $C = p/2$ in the complex convexity estimate

$$\Big(\int_0^{2\pi} |w - ae^{i\theta}|^p \,\frac{d\theta}{2\pi}\Big)^{1/p} \geq \Big(|w|^2 + \frac{p}{2}\, a^2\Big)^{1/2} , \quad w \in C \ , a \in R^+ ,$$

as conjectured by W.J. Davis, D.J.H. Garling and N. Tomczak-Jaegermann in 1984.

Camil Muscalu (Cornell University)

The Helicoidal Method

Abstract: The goal of the talk is to describe some of the aspects of the "helicoidal method" that we developed with Cristina Benea over the last few years. One particular more recent consequence of it, that we plan to discuss in some detail, is the proof of multiple vector valued, mixed-norm estimates for various (multi-linear) operators. Such estimates, which at this level of generality are new even in the case of paraproducts, seem to appear quite naturally in non-linear PDE contexts.

Kenji Nakanishi (Kyoto University)

Global dynamics and Strichartz estimate of the 4D Zakharov system below the ground state energy

Abstract: I would like to report our result on the global Strichartz estimate and scattering/blow-up dichotomy of Kenig-Merle type below the ground state energy for the Zakharov system in four space dimensions in the radial energy space. This is joint work with Zihua Guo, initially started from intensive discussions including Ioan Bejenaru and Sebastian Herr during the 2014 trimester program. The Zakharov system is not scaling invariant, but exhibits various critical phenomena in the 4D energy space, at least in the technical sense. The easiest way to observe the criticality is the subsonic limit sending the system to the cubic nonlinear Schrödinger equation, which is energy-critical by scaling. However, the most difficulty comes from the free wave component in $L^2(\mathbf{R}^4)$, which is removed in the subsonic limit. In order to treat that, we derive a Strichartz estimate for the Schrödinger equation with a free wave potential below the ground state L^2 norm, which is global and uniform with respect to the L^2 norm of potential, using the profile decomposition, the normal form, and the radial improvement in Strichartz estimate. Using the uniform Strichartz estimate, we are able to prove the global well-posedness, as well as the scattering versus (weak) blow-up result.

Tadahiro Oh (The University of Edinburgh)

On dispersive PDEs with singular random data

Abstract: A central theme in the study of "singular" nonlinear PDEs is "how to impose a structure" on a class of functions under consideration so that we can provide a proper meaning to a given nonlinearity.
In this talk, I will go over some of the recent development in stochastic dispersive PDEs, broadly interpreted with random data and/or stochastic forcing, and discuss how to impose structures (beyond specifying function spaces) to tackle these problems. I will also discuss how to interpret a classically ill-posed solution map by factorizing it into two steps.

Diogo Oliveira e Silva (University of Birmingham)

Sharp Strichartz inequalities for fractional and higher order Schrödinger equations

Abstract: It has long been understood that Strichartz estimates for the homogeneous Schrödinger equation correspond to adjoint Fourier restriction estimates on the paraboloid. The study of extremizers and sharp constants for the corresponding inequalities has a short but rich history. In this talk,

I will summarize it briefly, and then specialize to the case of certain planar power curves. A geometric comparison principle for convolution measures can be used to establish the corresponding sharp Strichartz inequality, and to decide whether extremizers exist. The mechanism underlying the possible lack of compactness is explained by the behaviour of extremizing sequences and will be described via concentration-compactness. Time permitting, I will show how this resolves a dichotomy from the recent literature concerning the existence of extremizers for the fourth order Schrödinger equation in one spatial dimension. This talk is based on joint work with Gianmarco Brocchi and René Quilodrán.

Angkana Rüland (Max Planck Institute for Mathematics in the Sciences)

On the Fractional Calderon Problem: Uniqueness, Stability and Reconstruction

Abstract: In this talk I discuss a nonlocal inverse problem, the fractional Calderón problem. This is an inverse problem for a fractional Schrödinger equation in which one seeks to recover information on an unknown potential by exterior measurements. In the talk, I prove uniqueness and stability of the "infinite data problem" and then address the recovery question. This also yields surprising insights on the uniqueness properties of the inverse problem in that it turns out that a single measurement suffices to uniquely recover the potential.
These properties are based on the very strong unique continuation and approximation properties of fractional Schrödinger operators, which are of independent interest and which I also discuss in the talk.

Birgit Schörkhuber (Karlsruhe Institute for Technology)

Self-similar blowup for the focusing energy-supercritical wave equation

Abstract: We review recent results on singularity formation for the focusing semilinear wave equation. In particular, we discuss the existence of non-trivial self-similar blowup solutions and their role in the time evolution of generic initial data. This talk is based on joint work with Irfan Glogić (University of Vienna).

Brian Street (University of Wisconsin-Madison)

Maximal Hypoellipticity

Abstract: In 1974, Folland and Stein introduced a generalization of ellipticity known as maximal hypoellipticity. This talk will be an introduction to this concept and some of the ways it generalizes ellipticity.

Michael Struwe (ETH Zürich)

Normalized harmonic map flow

Abstract: Finding non-constant harmonic 3-spheres for a closed target manifold N is a prototype of a super-critical variational problem. In fact, the direct method fails, as the infimum of the Dirichlet energy in any homotopy class of maps from the 3-sphere to any closed N is zero; moreover, the harmonic map heat flow may blow up in finite time, and even the identity map from the 3-sphere to itself is not stable under this flow.

To overcome these difficulties, we propose the normalized harmonic map flow as a new tool, and we show that for this flow the identity map from the 3-sphere to itself now, indeed, is stable; moreover, the flow converges to a harmonic 3-sphere also when we perturb the target geometry.
While our results are strongest in the perturbative setting, we also outline a possible global theory.

Luis Vega (BCAM-UPV/EHU)

Some lower bounds for solutions of Schrodinger evolutions

Abstract: I'll give some lower bounds for regular solutions of Schrodinger equations with bounded and time dependent complex potentials. Assuming that the solution has some positive mass at time zero within a ball of certain radius, we prove that this mass can be observed if one looks at the solution and its gradient in space-time parabolic regions outside of that ball. This is a joint work with Mikel Agirre.

Alexander Volberg (Michigan State University)

Poincaré inequalities on Hamming cube: analysis, combinatorics, probability

Abstract: We improve the constant $\frac{\pi}{2}$ in L^1-Poincaré inequality on Hamming cube. For Gaussian space the sharp constant in L^1 inequality is known, and it is $\sqrt{\frac{\pi}{2}}$ (Maurey–Pisier). For Hamming cube the sharp constant is not known, and $\sqrt{\frac{\pi}{2}}$ gives an estimate from below for this sharp constant. On the other hand, L. Ben Efraim and F. Lust-Piquard have shown an estimate from above: $C_1 \leq \frac{\pi}{2}$. There are at least two other proofs of the same estimate from above (we write down one of them in this note). Since those proofs are very different from the proof of Ben Efraim and Lust-Piquard but gave the same constant, that might have indicated that constant is sharp. But here we give a better estimate from above, showing that C_1 is strictly smaller than $\frac{\pi}{2}$. It is still not clear whether $C_1 > \sqrt{\frac{\pi}{2}}$. We discuss this circle of questions.

Jim Wright (University of Edinburgh)

On a higher dimensional version of the Benjamin–Ono equation

Abstract: We consider a higher dimensional version of the Benjamin–Ono equation, $\partial_t - R_1\Delta u + u\partial u_{x_1} u = 0$, where R_1 denotes the Riesz transform with respect to the first coordinate. We first establish space–time estimates for the associated linear equation, many of which are sharp. These estimates enable us to show that the initial value problem for the nonlinear equation is locally well-posed in L^2-Sobolev spaces $H^s(R^d)$, with $s > 5/3$ if $d = 2$ and $s > d/2 + 1/2$ if $d \geq 3$. We also provide ill-posedness results. This is joint work with Felipe Linares, Oscar G. Riaño and Keith M. Rogers.

Po-Lam Yung (The Chinese University of Hong Kong)

A proof of decoupling inequalities for the moment curve in 3 dimensions inspired by efficient congruencing

Abstract: In 2014 Wooley gave a proof of the cubic case of Vinogradov mean value theorem. The proof was subsequently simplified by Heath-Brown. In 2015, Bourgain, Demeter and Guth gave a proof of the general case of Vinogradov mean value theorem, by establishing decoupling inequalities for the moment curve in all dimensions. In this talk, we will discuss a decoupling interpretation of

efficient congruencing in 3 dimensions. In particular, inspired by the work of Heath-Brown, we will give a new proof of an $\ell^4 L^{12}$ decoupling inequality for the moment curve in 3 dimensions. This is joint work with Shaoming Guo and Zane Li, and is based in turn on earlier work of Zane Li in 2 dimensions.

Pavel Zorin-Kranich (University of Bonn)

Variational and jump inequalities

Abstract: Variational norms are parametrization-invariant versions of Hölder norms. They appear in the theory of rough paths and can also be used to quantify various convergence results, e.g. for truncated singular integrals and ergodic averages. Endpoint versions of such quantified results can be formulated using jump norms. I will present several endpoint results involving jump norms for diffusion semigroups, ergodic averages, and stochastic integrals from joint works with M. Mirek and E. Stein and with V. Kovač
