HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Winter School on

"Connections between representation theory and geometry"

October 5-9, 12-16 and 19-23, 2020

Abstracts for Mini Courses

Bernhard Keller (Université de Paris, Paris 7)

Introduction to A-infinity structures

Abstract: In this minicourse, we will present basic results on A-infinity algebras, their modules and their derived categories. We will start with two motivating problems from representation theory. Then we will briefly present the topological origin of A-infinity structures. We will then define and study A-infinity algebras and their morphisms. Of central importance are the bar construction and Kadeishvili's theorem on the existence of minimal models. We will then define the derived category of an A-infinity algebra or category and describe its full subcategory generated by the representables using twisted objects.

James Pascaleff (University of Illinois)

Introduction to Fukaya categories

Abstract: This minicourse will provide an introduction to Fukaya categories. I will assume that participants are also attending Keller's course on A_∞ categories.

- Lecture 1: Basics of symplectic geometry for Fukaya categories. Symplectic manifolds; Lagrangian submanifolds; exactness conditions; almost complex structures; holomorphic maps; Maslov indices and gradings.
- Lecture 2: Floer cohomology and the Fukaya category. Lagrangian intersection theory; Floer differential; A_∞ operations; Gromov compactification and A_∞ equations.
- Lecture 3: Examples of Fukaya categories. The case of surfaces; other cases as time permits.

Sibylle Schroll (University of Leicester)

A geometric model for the bounded derived category of a gentle algebra

Abstract: Gentle algebras are quadratic monomial algebras whose representation theory is well understood. In recent years they have played a central role in several different subjects such as in cluster algebras where they occur as Jacobian algebras of quivers with potentials obtained from triangulations of marked surfaces and in the context of homological mirror symmetry where graded

gentle algebras with zero differential appear in the construction of partially wrapped Fukaya categories of surfaces with stops. In these lectures we will recall the construction of a geometric model for the bounded derived category of an (ungraded) gentle algebra. We will see how the gentle algebra encodes not only a marked surface but also a line field on the surface. This line field allows to define a complete derived invariant for gentle algebras which generalises and completes the well-known derived invariant by Avella-Alaminos and Geiss. We will give explicit examples relating the introduced geometric model with the description of the partially wrapped Fukaya category in the work of Haiden, Katzarkov and Kontsevich.

Abstracts for Single Talks

Claire Amiot (Université Joseph Fourier)

Skew-gentle algebras and surface orbifolds

Abstract: In the 80's, Reiten and Riedtmann introduced the notion of skew-group algebra attached to an algebra with the action of a group by automorphisms. They studied in particular its representation theory. Skew-gentle algebras were introduced by Geiss and de la Peña in the 90's as certain skew-group algebras of gentle algebras. The aim of this talk is to show how the geometric model of the derived category of gentle algebras developed by Opper, Plamondon and Schroll can be adapted to skew-group algebras.
This is a joint work with Thomas Brüstle.

Tobias Dyckerhoff (Universität Hamburg)

Perverse sheaves and schobers on Riemann surfaces

Abstract: Reporting on joint work in progress with M. Kapranov, V. Schechtman, and Y. Soibelman, I will explain how to describe the derived constructible category of a stratified Riemann surface as representations of the so-called paracyclic category of the surface. This allows for geometric depictions of the various t-structures of interest (including the perverse one) and their interplay with Verdier duality. We will then discuss how this leads to an approach to categorified perverse sheaves (perverse schobers) and provide applications.

Fabian Haiden (University of Oxford)

From Hall algebras to legendrian skein algebras

Abstract: A mysterious relation between Hall algebras of Fukaya categories of surfaces and skein algebras was suggested by recent work of Morton-Samuelson and Samuelson-Cooper. I will discuss how this relation can be made precise using knot theory of legendrian curves and general gluing properties of skein and Hall algebras. Along the way I aim to motivate and review notions such as Hall algebras, Fukaya categories of surfaces, and skein theory. Based on arXiv:1908.10358, arXiv:1910.04182, and ongoing work with Ben Cooper.

Gustavo Jasso (Universität Bonn)

Partially wrapped Fukaya categories of symmetric products of marked disks

Abstract: Partially wrapped Fukaya categories of symmetric products of marked surfaces were introduced by Auroux so as to give a symplecto-geometric intepretation of the bordered Heegaard-Floer homology of Lipshitz, Ozsváth and Thurston. In this talk, I will explain the equivalence between the partially wrapped Fukaya categories of symmetric products of marked disks and the perfect derived categories of a (discrete) family of finite-dimensional algebras introduced by Iyama, the so-called higher Auslander algebras of type A. In particular, I will explain some of the rich structure present among these mathematical objects as they also relate to the higher Waldhausen S-constructions of Dyckerhoff and Poguntke.
This is a report with Tobias Dyckerhoff and Yanki Lekili and also contains work with Dyckerhoff and Walde.

Ailsa Keating (University of Cambridge)

Homological mirror symmetry for log Calabi-Yau surfaces

Abstract: Given a log Calabi-Yau surface Y with maximal boundary D, I'll explain how to construct a mirror Landau-Ginzburg model, and sketch a proof of homological mirror symmetry for these pairs when (Y,D) is distinguished within its deformation class (this is mirror to an exact manifold). I'll explain how to relate this to the total space of the SYZ fibration predicted by Gross–Hacking–Keel, and, time permitting, explain ties with earlier work of Auroux–Katzarkov–Orlov and Abouzaid. Joint work with Paul Hacking.

Yanki Lekili (King's College London)

Homological mirror symmetry for not-so-simple singularities

Abstract: In a joint work with Ueda, we outlined a set of explicit conjectures that precisely explain how homological mirror symmetry should work for Milnor fibers of invertible polynomials. We can prove these conjectures in a number of interesting cases such as the case of simple singularities (in any dimension), when the corresponding categories turns out to be equivalent to the derived category of the Calabi-Yau completion of the path algebra of Dynkin quivers. In general, one has to consider global deformations, but we can show that the possible deformations are parametrized by a moduli space of A_∞ structures which we identify with a certain moduli space of polarized Calabi-Yau varieties. I will explain the case of the Milnor fiber of $x^4 + y^4 + z^4$ (and its higher dimensional generalizations) which is among the cases of not-so-simple singularities for which we can fully rigorously prove homological mirror symmetry.

Ivan Smith (University of Cambridge)

Plumbings and flops

Abstract: I will discuss geometric models, in both symplectic and algebraic geometry, for the A-infinity algebras associated to potentials on the oriented two-cycle quiver; the models show an interesting dependence on the characteristic of the ground field. This talk reports on joint work with Michael Wemyss.
