Workshop on
“Continuous approaches to discrete optimization”
October 11 - 15, 2021
organized by
Chandra Chekuri (Illinois), Daniel Dadush (Amsterdam), Yin Tat Lee (Washington),
Stephen Wright (Wisconsin)

Abstracts

Monday, October 11th

Jan van den Brand (Simons Institute, Berkeley)
From Interior Point Methods to Data Structures and back
Abstract: Linear Programs (LPs) capture many optimization problems such as shortest paths or bipartite matching. In the past years, there have been substantial improvements for LP solvers resulting in algorithms that run in nearly linear time for dense LPs. This also led to a nearly linear time algorithm for bipartite matching on dense graphs. In this talk I will explain how these improvements came to be from an interplay of interior point methods and dynamic algorithms (data structures).

Rasmus Kyng (ETH)
A numerical analysis approach to convex optimization
Abstract: In convex optimization, we can usually obtain $O(1)$-approximate solutions much faster than high accuracy $(1 + 1/poly(n))$-approximate solutions. One major exception is L2-regression, where low accuracy algorithms can be converted into high-accuracy ones via iterative refinement. I will present generalizations of iterative refinement to p-norms, which lead to high-accuracy algorithms based on crudely solving only a polylogarithmic number of residual problems. I will also discuss several results that build on this new approach, including p-norm regression using $m^{1/3}$ linear system solves, and p-norm flow in undirected unweighted graphs in almost-linear time.

Yang Liu (Stanford University)
Fully Dynamic Maximum Flows: Sparse Maxflow Faster than Goldberg-Rao
Abstract: We give an algorithm for computing exact maximum flows on graphs with $m$ edges and integer capacities in the range $[1, U]$ in $O(m^{3/2} \log^2 U)$ time. For sparse graphs with polynomially...
bounded integer capacities, this is the first improvement over the \( \tilde{O}(m^{1.5} \log U) \) time bound from [Goldberg-Rao JACM ’98].

Our algorithm revolves around dynamically maintaining the augmenting electrical flows at the core of the interior point method based algorithm from [Madry JACM ’16]. This entails designing data structures that, in limited settings, return edges with large electric energy in a graph undergoing resistance updates.

Debmalya Panigrahi (Duke University)

The Isolating Cuts Lemma: A new tool for solving minimum cut problems

Abstract: Minimum cut problems are among the most well-studied questions in combinatorial optimization. In this talk, I will introduce a simple but powerful new tool for solving minimum cut problems called the isolating cuts lemma. I will show how this tool can be employed to obtain faster algorithms for several fundamental min-cut problems, namely global min-cut, Steiner min-cut, and all-pairs min-cut. For these problems, the new results represent the first improvement in their runtimes in several decades.

Sorrachai Yingchareonthawornchai (Aalto University)

Approximating k-Edge-Connected Spanning Subgraphs via a Fast Linear Program Solver

Abstract: In the k-edge-connected spanning subgraph (kECSS) problem, our goal is to compute a minimum-cost sub-network that is resilient against up to k link failures: Given an n-node m-edge graph with cost function on edges, we want to compute the minimum-cost k-edge-connected spanning subgraph. This problem generalizes the minimum spanning tree problem and is the uniform case of the large class of survival network design problems SNDP. The whole SNDP class admits 2-approximation algorithms. Our concern is how fast an algorithm can achieve this approximation factor for kECSS. Previously, (Khuller and Vishkin, STOC’92) presented a 2-approximation algorithm that runs in time \( O(mnk) \). The running time was improved to \( O(n^2) \) with the tradeoff in the approximation guarantee of \( (2k - 1) \) (Gabow, Goemans, and Williamson, IPCO’93). In this talk, I explain a new algorithm that improves the running time of both aforementioned algorithms while keeping the approximation ratio arbitrarily close to two. Our main contribution is an algorithm that \( (1 + \epsilon) \)-approximates the optimal fractional solution (and therefore \( (2 + \epsilon) \)-approximates the optimal solution cost) in \( \tilde{O}(mk/\epsilon^3) \) time, which is nearly-linear when \( k = \text{polylog}(n) \). Such a fractional solution can be turned into a \( (2 + \epsilon) \) approximation algorithm that runs in time \( \tilde{O} \left( \frac{km}{\epsilon^2} + \frac{k^2n^{1.5}}{\epsilon^3} \right) \) for (integral) kECSS. This is joint work with Parinya Chalermsook, Chien-Chung Huang, Danupon Nanongkai, Thatchaphol Saranurak, and Pattara Sukprasert.

Kent Quanrud (University of Illinois Urbana-Champaign)

On Iterative Peeling and Supermodularity for Densest Subgraph

Abstract: The densest subgraph problem in a graph (DSG), in the simplest form, is the following. Given an undirected graph \( G = (V,E) \) find a subset \( S \subseteq V \) of vertices that maximizes the ratio \( |E(S)|/|S| \) where \( E(S) \) is the set of edges with both endpoints in \( S \). DSG and several of its variants are well-studied in theory and practice and have many applications in data mining and network analysis.

Greedy peeling algorithms have been very popular for DSG and several variants due to their efficiency, empirical performance, and worst-case approximation guarantees. Recently, Boob et al. developed an
iterative peeling algorithm for DSG which appears to work very well in practice, and made a conjecture about its convergence to optimality. We affirmatively answer their conjecture as well as extensions of the conjecture for some supermodular extensions of DSG. This talk will primarily be about this result and in particular highlight the interesting connections we learned along the way. This is joint work with Chandra Chekuri and Manuel R. Torres.

Tuesday, October 12th

**Sally Dong** (University of Washington)

**Nested Dissection Meets IPMs: Planar Min-Cost Flow in Nearly-Linear Time**

**Abstract:** We present a nearly-linear time algorithm for finding a minimum cost flow in planar graphs with polynomially bounded integer costs and capacities. Previously, the fastest algorithm for this problem was based on interior point methods (IPMs) and works for general sparse graphs in $O(n^{1.5} \text{polylog} n)$ time [Daitch-Spielman, STOC' 08].

**Bento Natura** (London School of Economics)

**Fast Exact Solvers for Linear Programs via Interior Point Methods**

**Abstract:** Recent years have seen tremendous progress in approximate solvers for Linear Programs (LP) based on Interior-Point Methods (IPM). In this talk we show how to leverage these algorithms to design algorithms that solve LPs exactly. The running time of these algorithms depends on the constraint matrix only. We will present these algorithms in two different regimes: In the first, we use approximate LP solvers in a blackbox manner, extending Tardos’s Framework (Oper. Res. ’86). In the second, we design an exact IPM, using ideas of the recent approximate IPMs to improve the running time. Based on joint work with Daniel Dadush, Sophie Huiberts and László Végh.

**Jacek Gondzio** (University of Edinburgh)

**Applying interior point algorithms in column generation and cutting plane methods**

**Abstract:** Advantages of interior point methods (IPMs) applied in the context of nondifferentiable optimization arising in cutting planes/column generation applications will be discussed. Some of the many false views of the combinatorial optimization community on interior point methods applied in this context will be addressed and corrected. In particular, IPMs deliver a natural stabilization when restricted master problems are solved and guarantee fast convergence, measured with merely a few master iterations needed to localize the solution. And IPMs can do warm-starts.

The talk will focus on key concepts and will use only gentle and intuitive “theoretical” arguments, it is worth mentioning that there exists several references which address the issues in more detail and a ready-to-use software which implements some of the presented techniques, the Primal-Dual Column Generation Method (PDCGM), all available at: [http://www.maths.ed.ac.uk/~gondzio/software/pdcgm.html](http://www.maths.ed.ac.uk/~gondzio/software/pdcgm.html).
Andrea Lodi (Cornell Tech)

Cutting Plane Generation Through Sparse Principal Component Analysis

Abstract: Quadratically-constrained quadratic programs (QCQPs) are optimization models whose remarkable expressiveness has made them a cornerstone of methodological research for nonconvex optimization problems. However, modern methods to solve a general QCQP fail to scale, encountering computational challenges even with just a few hundred variables. Specifically, a semidefinite programming (SDP) relaxation is typically employed, which provides strong dual bounds for QCQPs, but relies on memory-intensive algorithms. An appealing alternative is to replace the SDP with an easier-to-solve linear programming relaxation, while still achieving strong bounds. In this work, we make advances towards achieving this goal by developing a computationally-efficient linear cutting plane algorithm that emulates the SDP-based approximations of nonconvex QCQPs. The cutting planes are required to be sparse, in order to ensure a numerically attractive approximation, and efficiently computable. We present a novel connection between such sparse cut generation and the sparse principal component analysis problem in statistics, which allows us to achieve these two goals. We show extensive computational results advocating for the use of our approach.

Jens Vygen (University of Bonn)

Continuous approaches to VLSI routing

Abstract: We survey the state of the art in VLSI routing, including continuous approaches such as fractional min-max resource sharing, continuous global routing, and goal-oriented shortest path search sped up by geometric distance queries. These are key components of BonnRoute, which is used for routing some of the most complex microprocessors in industry.

Robert Luce (Gurobi)

Solving nonconvex quadratic optimization problems with Gurobi

Abstract: We review some of the key techniques Gurobi uses to solve nonconvex quadratic optimization problems to global optimality. In particular we will discuss the McCormick relaxation, powerful cutting planes in this context, and local heuristics.

Aaron Sidford (Stanford University)

Unit Capacity Maximum Flow in Almost $m^{4/3}$ Time

Abstract: The maximum flow problem on unit capacity graphs is a fundamental problem in combinatorial optimization with multiple applications including computing the maximum number of disjoint paths between a pair of vertices in a graph and computing a maximum cardinality matching in a bipartite graph. In this talk I will present recent advances in solving this problem. In particular, I will discuss how recent advances in solving mixed l2-lp flows can be coupled with interior point methods to improve the running time for this problem, culminating in a runtime of almost $m^{4/3}$. This talk reflects joint work with Yang P. Liu and Tarun Kathuria.
Bridging the Gap Between Tree and Connectivity Augmentation: Unified and Stronger Approaches

**Abstract:** The Connectivity Augmentation Problem (CAP) is one of the most basic survivable network design problems. The task is to increase the edge-connectivity of a graph $G$ by one unit by adding a smallest number of additional edges from a given set. If the edge-connectivity of $G$ is odd, CAP reduces to a heavily studied special case known as the Tree Augmentation Problem (TAP). Despite significant progress on TAP, only very recently, Byrka, Grandoni, and Ameli (STOC 2020) managed to obtain an algorithm for CAP with approximation guarantee better than 2 by presenting a 1.91-approximation based on techniques disjoint from recent TAP advances.

In this talk, we will present new methods that allow for leveraging insights and techniques from TAP to approach CAP. Combined with a novel analysis technique, we obtain a 1.393-approximation for CAP. This significantly improves in a unified way on the previously best approximation factor for CAP (1.91) and also TAP (1.458).

This is joint work with Federica Cecchetto.

---

Matthias Mnich (TU Hamburg)

**Approximation Algorithms for Hard Cut Problems via Continuous Relaxations**

**Abstract:** We show new approximation algorithms for some hard cut problems on graphs, using methods from continuous optimization.

---

Sebastian Pokutta (Zuse Institute, TU Berlin)

**Fast Algorithms for Packing Proportional Fairness and its Dual**

**Abstract:** The proportional fair resource allocation problem is a major problem studied in flow control of networks, operations research, and economic theory, where it has found numerous applications. This problem, defined as the constrained maximization of $\sum_i \log x_i$, is known as the packing proportional fairness problem when the feasible set is defined by positive linear constraints and $x \geq 0$. In this work, we present a distributed accelerated first-order method for this problem which improves upon previous approaches. We also design an algorithm for the optimization of its dual problem. Both algorithms are width-independent.

(joint work with: Francisco Criado and David Martínez-Rubio)

---

Stefan Weltge (TU Munich)

**Simple Iterative Methods for Linear Optimization over Convex Sets**

**Abstract:** We consider the problem of maximizing a linear functional over a general convex body $K$ given by a separation oracle. While the standard cutting plane algorithm performs well in practice, no bounds on the number of oracle calls can be given. In contrast, methods with strong theoretical guarantees, such as the ellipsoid method, are often impractical. We give a new simple method with weaker theoretical bounds but that performs considerably better in practice.
It is based on classical Von Neumann and Frank-Wolfe algorithms, and requires $O(R^4/(r^2\epsilon^2))$ calls to the oracle to find an additive $\epsilon$-approximate solution, where it is assumed that $K$ contains a ball of radius $r$ and is contained inside the origin centered ball of radius $R$. If the inner $r$-ball is centered at the origin, we give a simplified variant that outputs a multiplicative $(1 + \epsilon)$-approximate solution using $O(R^2/(r^2\epsilon^2))$ oracle calls.

We evaluate our method on instances from combinatorial optimization, semidefinite programming, and machine learning. In terms of oracle calls, we observe that it is comparable to the standard cutting plane method and often even faster.

This is joint work with Daniel Dadush, Christopher Hojny, and Sophie Huiberts.

Zhao Song (Adobe Research)

Fast Iterative Algorithm via Nearest/Furthest Neighbor Search

Abstract: We show how to use nearest/furthest neighbor search data-structure to spend-up the cost per iteration of a number of iterative algorithms in many fields, reinforcement learning, discrepancy algorithms, and deep learning.

Alina Ene (Boston University)

Adaptive gradient descent methods for constrained optimization

Abstract: Adaptive gradient descent methods, such as the celebrated Adagrad algorithm (Duchi, Hazan, and Singer; McMahan and Streeter) and ADAM algorithm (Kingma and Ba), are some of the most popular and influential iterative algorithms for optimizing modern machine learning models. Algorithms in the Adagrad family use past gradients to set their step sizes and are remarkable due to their ability to automatically adapt to unknown problem structures such as (local or global) smoothness and convexity. However, these methods achieve suboptimal convergence guarantees even in the standard setting of minimizing a smooth convex function, and it has been a long-standing open problem to develop an accelerated analogue of Adagrad in the constrained setting.

In this talk, we present one such accelerated adaptive algorithm for constrained convex optimization that simultaneously achieves optimal convergence in the smooth, non-smooth, and stochastic setting, without any knowledge of the smoothness parameters or the variance of the stochastic gradients.

The talk is based on joint work with Huy Nguyen (Northeastern University) and Adrian Vladu (CNRS & IRIF, University de Paris).

Jelena Diakonikolas (University of Wisconsin-Madison)

Local Acceleration of Frank-Wolfe Methods

Abstract: Conditional gradients (a.k.a. Frank-Wolfe) methods are the convex optimization methods of choice in settings where the feasible set is a convex polytope for which projections are expensive or even computationally intractable, but linear optimization can be implemented efficiently. Unlike projection-based methods, Frank-Wolfe methods provably cannot attain globally accelerated rates of convergence in the sense of Nesterov. We show that, however, local acceleration of Frank-Wolfe methods is possible. Namely, we show that for a class of smooth strongly convex objectives we can construct a method that does not require expensive projections onto the polytope and, following a finite burn-in time that is independent of the target error, attains an accelerated convergence rate, scaling with the square-root condition number of the objective function. We further show that such a
method can be implemented without the knowledge of the problem parameters such as smoothness, strong convexity, or the diameter of the feasible set.

Thursday, October 14th

**Roie Levin** (Carnegie Mellon University)

**Random Order Set Cover is as Easy as Offline**

**Abstract:** We give a polynomial time algorithm for Online Set Cover with a competitive ratio of $O(\log mn)$ when the elements are revealed in random order, essentially matching the best possible offline bound of $O(\log n)$ and circumventing the $O(\log m \log n)$ lower bound known in adversarial order. We also extend the result to solving pure covering IPs when constraints arrive in random order. The algorithm is a multiplicative-weights-based round-and-solve approach we call Learn-Or-Cover. We maintain a coarse fractional solution that is neither feasible nor monotone increasing, but can nevertheless be rounded online to achieve the claimed guarantee (in the random order model). This gives a new offline algorithm for Set Cover that performs a single pass through the elements, which may be of independent interest. This is joint work with Anupam Gupta and Gregory Kehne.

**Gerard Cornuejols** (Carnegie Mellon University)

**Dyadic linear programming**

**Abstract:** A finite vector is dyadic if each of its entries is a dyadic rational number, i.e. if it has an exact floating point representation. We study the problem of finding a dyadic optimal solution to a linear program, if one exists. This is joint work with Ahmad Abdi, Bertrand Guenin and Levent Tunec.

**Ola Svensson** (EPFL)

**Learning-Augmented Online Algorithms and the Primal-Dual Method**

**Abstract:** The design of learning-augmented online algorithms is a new and active research area. The goal is to understand how to best incorporate predictions of the future provided e.g. by machine learning algorithms that rarely come with guarantees on their accuracy. In the absence of guarantees, the difficulty in the design of such learning-augmented algorithms is to find a good balance: on the one hand, following blindly the prediction might lead to a very bad solution if the prediction is misleading. On the other hand, if the algorithm does not trust the prediction at all, it will simply never benefit from an excellent prediction. An explosion of recent results solve this issue by designing smart algorithms that exploit the problem structure to achieve a good trade-off between these two cases. In this talk, we will discuss this emerging line of work. In particular, we will show how to unify and generalize some of these results by extending the powerful primal-dual method for online algorithms to the learning augmented setting. This is joint work with Etienne Bamas and Andreas Maggiori.
Anupam Gupta  (Carnegie Mellon University)

Covering LP Relaxations for k-Server

Abstract: We give a covering LP relaxation for k-server, and use it to obtain a polylogarithmic (currently in n and Delta, the aspect ratio) for k-server. (This relaxation extends to the problem with time-windows.) In this talk we discuss this LP relaxation, and the main ideas behind the online primal-dual analysis.
Joint work with Amit Kumar and Debmalya Panigrahi.

Sebastian Bubeck  (Microsoft Research)

Chasing Small Sets

Abstract: I will present an approach based on mirror descent (with a time-varying multiscale entropy functional) to chase small sets in arbitrary metric spaces. This could in particular resolve the randomized competitive ratio of the layered graph traversal problem introduced by Papadimitriou and Yannakakis in 1991.

Friday, October 15th

Haotian Jiang  (University of Washington)

Minimizing Convex Functions with Integral Minimizers

Abstract: Given a separation oracle $SO$ for a convex function $f$ that has an integral minimizer inside a box with radius $R$, we show how to find an exact minimizer of $f$ using at most

- $O(n(n + \log(R)))$ calls to $SO$ and $\text{poly}(n, \log(R))$ arithmetic operations, or
- $O(n \log(nR))$ calls to $SO$ and $\exp(O(n)) \cdot \text{poly}(\log(R))$ arithmetic operations.

When the set of minimizers of $f$ has integral extreme points, our algorithm outputs an integral minimizer of $f$. This improves upon the previously best oracle complexity of $O(n^2(n + \log(R)))$ for polynomial time algorithms obtained by [Grötschel, Lovász and Schrijver, Prog. Comb. Opt. 1984, Springer 1988] over thirty years ago.

For the Submodular Function Minimization problem, our result immediately implies a strongly polynomial algorithm that makes at most $O(n^3)$ calls to an evaluation oracle, and an exponential time algorithm that makes at most $O(n^2 \log(n))$ calls to an evaluation oracle. These improve upon the previously best $O(n^3 \log^2(n))$ oracle complexity for strongly polynomial algorithms given in [Lee, Sidford and Wong, FOCS 2015] and [Dadush, Végh and Zambelli, SODA 2018], and an exponential time algorithm with oracle complexity $O(n^3 \log(n))$ given in the former work.

Our result is achieved via a reduction to the Shortest Vector Problem in lattices. Our analysis of the oracle complexity is based on a potential function that captures simultaneously the size of the search set and the density of the lattice, which we analyze via convex geometry tools. Paper available at https://arxiv.org/abs/2007.01445.
A polynomial lower bound on the number of rounds for efficient submodular function minimization

Abstract: Submodular function minimization (SFM) is a fundamental discrete optimization problem generalizing problems such as minimum cut problems in graphs and hypergraphs, and the matroid intersection problem. It is well-known and remarkable that a submodular function on an N-element universe can be minimized using only poly(N) evaluation queries. However, all such efficient algorithms have poly(N) rounds of adaptivity. A natural question is whether one can get efficient parallel SFM algorithms with much fewer rounds of adaptivity. Recently, in STOC 2020, Balkanski and Singer proved an \( \Omega(\log N / \log \log N) \) lower bound on the rounds of adaptivity. However, it left open the possibility of a polylogarithmic depth efficient algorithm; such algorithms are indeed known for efficient (approximate) submodular function *maximization* versions.

In this talk, I will show that submodular function minimization is not friendly to parallelization. In particular, I’ll sketch a proof that any SFM algorithm making at most poly(N) calls, must proceed in at least (roughly) \( N^{1/3} \) rounds. This is joint work with Yu Chen and Sanjeev Khanna, both of whom are at the University of Pennsylvania.