Conference on
“Dynamics on homogeneous spaces”
March 16-20, 2020
organized by
Manfred Einsiedler (Zürich), Martin Möller (Frankfurt), Anke Pohl (Bremen), Tom Ward (Leeds)

Abstracts

Nicolas de Saxcé (CNRS - Université Paris 13, LAGA)

Spaces of lattices and diophantine approximation
Abstract: We shall give an overview of different results in diophantine approximation that can be understood from the behavior of diagonal orbits in some spaces of lattices.

Daniel El-Baz (TU Graz)

Effective equidistribution of primitive rational points on expanding horospheres
Abstract: In a 2016 paper, Manfred Einsiedler, Shahar Mozes, Nimish Shah and Uri Shapira used techniques from homogeneous dynamics to establish the equidistribution of primitive rational points on expanding horospheres in the space of unimodular lattices in at least 3 dimensions. Due to the nature of their proof, relying in particular on Marina Ratner’s measure-classification theorem, their result does not come with a quantitative error term. I will discuss a joint work with Bingrong Huang and Min Lee, in which we pursue an analytic number-theoretic approach to give a rate of convergence for a specific horospherical subgroup in any dimension. This extends work of Min Lee and Jens Marklof who dealt with the 3-dimensional case in 2018.

Weikun He (The Hebrew University of Jerusalem)

Equidistribution of linear random walks on the torus
Abstract: Quantitative equidistribution for linear random walks on the torus was first obtained by Bourgain, Furman, Lindenstrauss and Mozes. In this talk I will present a recent progress where the proximality assumption in their result is relaxed. This is a joint work with Nicolas de Saxcé.
Asaf Katz (University of Chicago)

An application of Margulis’ inequality to effective equidistribution

Abstract: Ratner’s celebrated equidistribution theorem states that the trajectory of any point in a homogeneous space under a unipotent flow is getting equidistributed with respect to some algebraic measure. In the case where the action is horospherical, one can deduce an effective equidistribution result by mixing methods, an idea that goes back to Margulis’ thesis. When the homogeneous space is non-compact, one needs to impose further “diophantine conditions” over the base point, quantifying some recurrence rates, in order to get a quantified equidistribution result. In the talk I will discuss certain diophantine conditions, and in particular I will show how a new Margulis’ type inequality for translates of horospherical orbits helps verify such conditions, leading to a quantified equidistribution result for a large class of points, akin to the results of A. Strombergsson dealing with {\text{SL}}_2 case. In particular we deduce a fully effective quantitative equidistribution statement for horospherical trajectories of lattices defined over number fields, without pertaining to Roth’s strong subspace theorem.

Osama Khalil (University of Utah)

Diophantine approximation on fractals and homogeneous flows

Abstract: The theory of Diophantine approximation is underpinned by Dirichlet’s fundamental theorem. Broadly speaking, the main questions in the theory concern quantifying the prevalence of points with exceptional behavior with respect to Dirichlet’s result. Badly approximable, very well approximable and Dirichlet-improvable points are among the most well-studied such exceptional sets. The work of Dani and Kleinbock-Margulis connects these questions to the recurrence behavior of certain flows on homogeneous spaces. After a brief overview, I will discuss new results giving a sharp upper bound on the Hausdorff dimension of divergent orbits of certain diagonal flows emanating from fractals on the space of lattices. Connections to theory of projections of self-similar measures will be presented.

Ilya Khayutin (Northwestern University)

Non-vanishing of class group $L$-functions

Abstract: Various $L$-functions can be computed as integrals of automorphic forms over a periodic orbit of a subgroup $H < G$ on arithmetic homogeneous spaces $\Gamma \backslash G$. Several methods to establish non-vanishing of these $L$-functions at the central point utilize one of two properties of the associated orbit: equidistribution or escape of mass. I will show how a combination of weak versions of both properties for Eisenstein series over toral packets lead to a surprisingly strong result for non-vanishing of class group $L$-function for number fields of small regulator and arbitrary degree.

Maxim Kirsebom (Universität Hamburg)

On a limiting distribution for maximal cusp excursions of the unipotent flow

Abstract: The unipotent flow on the unit tangent bundle of the modular surface is a classic example of a homogeneous flow when understood through the identification with $\text{PSL}_2(R)/\text{PSL}_2(Z)$. The ergodicity of the flow implies that almost every orbit is dense in the space and hence must eventually make excursions deeper and deeper into the cusp. We are interested in understanding the nature of
these excursions. In the described setting, and more generally, Athreya and Margulis proved that the maximal excursions obey the logarithm law almost surely, meaning that their growth rate scales the logarithm of the time. In this work we focus on a more precise description of this behaviour, namely determining the probability that the deepest excursion fails to outperform the expected asymptotic behaviour by an additive amount. This question may be phrased in the language of extreme value statistics and we establish some results towards a complete extreme value law in this setting. The methods used are based on classical ideas from geometry of numbers. This is work in progress, joint with Keivan Mallahi-Karai.

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**Manuel Luethi (ETH Zürich)**

**Joinings of diagonal flows and application to elliptic curve**

**Abstract:** I will shortly discuss the classification of joinings for certain diagonal actions by Einsiedler and Lindenstrauss and some of its applications. Then I will detail the following recent application to elliptic curves: The simultaneous reduction with respect to several distinct inert primes of the set of complex multiplication curves of discriminant $D$ is surjective on the set of isomorphism classes of supersingular curves, as $-D$ goes to infinity under two “external” congruence conditions. This is a joint work with Menny Aka, Philippe Michel, and Andreas Wieser.

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**Taylor McAdam (Yale University)**

**Almost-prime times in horospherical flows**

**Abstract:** In applications to number theory, it is often desirable for dynamical results to be effective (that is, bounds are given for rate of convergence for dynamical phenomena). In this talk, I will describe an effective result for horospherical flows on the space of lattices using the “thickening” method of Margulis, and use this result to provide an effective rate for the equidistribution of arithmetic progressions in horospherical flows using a method of Venkatesh. I will then describe an application to studying the distribution of almost-prime times (i.e. integer times with fewer than a fixed number of prime factors) in horospherical orbits and discuss connections of this work to Sarnak’s Mobius disjointness conjecture.

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**Wenyu Pan (University of Chicago)**

**Exponential mixing of geodesic flow for geometrically finite manifolds with cusps**

**Abstract:** Let $H^n$ be the hyperbolic $n$-space and $\Gamma$ be a geometrically finite discrete subgroup in $\text{Isom}_+(H^n)$ with cusps. In forthcoming joint work with Jialun Li, we establish exponential mixing of geodesic flow on the unit tangent bundle $T^1(\Gamma\backslash H^n)$. Previously, such results were proved by Mohammadi-Oh and Edwards-Oh for $\Gamma$ with large critical exponent and Stoyanov for convex cocompact discrete subgroups. We obtain our result by constructing a nice coding for the geodesic flow, which in particular satisfies the exponential tail condition, and then proving a spectral bound on transfer operator building on Dolgopyat’s framework. The construction of the coding was partly inspired by the works of Lai-Sang Young and Burns-Masur-Matheus-Wilkinson.
Nattalie Tamam (University of California, San Diego)

Effective equidistribution of horospherical flows in infinite volume

Abstract: The horospherical flow on finite-volume hyperbolic surfaces is well-understood. In particular, effective equidistribution of non-closed horospherical orbits is known. In this talk, we will explore the infinite volume setting. This is joint work with Jacqueline Warren.

Pengyu Yang (ETH Zürich)

Equidistribution of expanding translates of curves in homogeneous spaces

Abstract: Let $G$ be a semisimple connected real algebraic group, and $\Gamma$ a lattice in $G$. We will show that in $G/\Gamma$, the expanding translates of a non-degenerate real-analytic curve under a flow get equidistributed with respect to the Haar measure. We will also discuss its application to Diophantine approximation on matrices. This extends earlier works by N. Shah and L. Yang. The proof relies on Ratner’s theorem on measure rigidity for unipotent flows, linearization technique, and a new ingredient coming from geometric invariant theory.