Lisa Beck (Universität Augsburg)

Regularization by noise for the stochastic transport equation

Abstract: We discuss several aspects of regularity and uniqueness for weak \((L^\infty-)\) solutions to the (deterministic and stochastic) transport equation

\[ du = b \cdot Du \, dt + \sigma Du \circ dW_t. \]

Here, \(b\) is a vector field (the drift), \(u\) is the unknown, \(\sigma\) is a real number, \(W_t\) is a Brownian motion, and the stochastic term is interpreted in the Statonovich sense. For the deterministic equation (\(\sigma = 0\)) it is well-known that multiple solutions may exist and that solutions may blow up from smooth initial data in finite time if the drift is not regular enough. For the stochastic equation (\(\sigma \neq 0\)) instead, it turns out that a suitable integrability condition (known from fluid dynamics as the Ladyzhenskaya–Prodi–Serrin condition) on the drift is sufficient to prevent the formation of non-uniqueness and of singularities. After a short review of some techniques for the deterministic equation we explain how this regularization phenomenon, namely the conservation of Sobolev regularity of the initial data and the restoration of uniqueness, is obtained by means of PDE techniques (as opposed to stochastic characteristics). The results presented in this talk are part of a joint project with F. Flandoli, M. Gubinelli and M. Maurelli.

Hakima Bessaih (University of Wyoming)

Stochastic homogenization for some porous media models

Abstract: We are investigating some models arising from heterogenous porous media when some parameters like permeability or porosity are defined in terms of a stochastic process which is solution of an SDE. Various scales are involved both in time and space that will depend on a parameter. We investigate the well-posedness of the coupled system and its limiting behavior when this parameter goes to zero. The limiting equation will involve the invariant measure associated to the stochastic process and some averaged quantities. Our focus will be on advection-diffusion and convection-diffusion models.
Modulation Equations for SPDEs on unbounded domains

Abstract: We consider the approximation via modulation equations for nonlinear stochastic partial differential equations (SPDEs) like the stochastic Swift-Hohenberg (SH) equation, which serves as a toy model for the convective instability in Rayleigh-Benard convection. Close to a bifurcation of a single mode a small band of infinitely many eigenvalues changes stability. Thus solutions of SH are well described by a modulated wave, where the amplitude solves a stochastic Ginzburg-Landau (GL) equation with space time white noise. In the one-dimensional case on the whole real line due to the weak regularity of solutions the standard deterministic methods for modulation equations fail, as we need weighted spaces that allow for unboundedness at infinity of solutions, which is natural for translation invariant noise. Moreover, solutions of GL are only Hölder-continuous, which gives just enough regularity to obtain the approximation result.

Evolution on random varying (combinatorial and metric) graphs

Abstract: Evolution equations on graphs are a well established topic in the literature. In this work, we consider a randomly varying environment and we discuss the stability and convergence to equilibrium of the resulting system, both in the combinatorial setting (the information is stored on the nodes) and the metric setting (the information is spread along the edges). Joint work with Delio Mugnolo (Fernuniversität Hagen) and Francesca Cottini (Milano Bicocca)

Stochastic Navier-Stokes equations on a thin spherical domain

Abstract: We consider the incompressible Navier-Stokes equations on a thin spherical domain $Q_{\varepsilon}$ along with free boundary conditions under a random forcing. We show that the martingale solution of these equations converge to the martingale solution of the stochastic Navier-Stokes equation considered on a sphere $S$ as the thickness converges to zero. This talk is based on a joint research with G. Dhariwal (Vienna) and Q. T. Le Gia (UNSW, Sydney).

Invariant measures for stochastic 2D damped Euler equations

Abstract: We consider the 2D damped Euler equations with additive noise. When the noise is smooth in space, we define a Markov semigroup in the space $L^\infty$ equipped with the weak-* topology and prove existence of invariant measures by means of Krylov-Bogoliubov’s method. Joint work with H. Bessaih.

The Kolmogorov equation associated to SPDEs: theoretical and numerical problems

Abstract: The present knowledge about solvability of Kolmogorov equations associated to SPDEs is advanced but incomplete, especially for what concerns SPDEs of fluid mechanics. Open questions exist
both theoretically, especially when the drift is unbounded; and numerically, since classical methods - different from Monte Carlo - do not work in high dimensions. We discuss both the open questions and first steps of solution. This is a joint research with Dejun Luo and Cristiano Ricci and it is performed using the grant PRIN 2015, Deterministic and Stochastic Evolution Equations.

Benjamin Gess (Max Planck Institute for Mathematics in the Sciences & Universität Bielefeld)

Large deviations for conservative, stochastic PDE and non-equilibrium fluctuations

Abstract: Macroscopic fluctuation theory provides a general framework for far from equilibrium thermodynamics, based on a fundamental formula for large fluctuations around (local) equilibria. This fundamental postulate can be informally justified from the framework of fluctuating hydrodynamics, linking far from equilibrium behavior to zero-noise large deviations in conservative, stochastic PDE. In this talk, we will give rigorous justification to this relation in the special case of the zero range process. More precisely, we show that the rate function describing its large fluctuations is identical to the rate function appearing in zero noise large deviations to conservative stochastic PDE, by means of proving the Gamma-convergence of rate functions to approximating stochastic PDE. The proof of Gamma-convergence is based on the well-posedness of the skeleton equation – a degenerate parabolic-hyperbolic PDE with irregular coefficients, the proof of which extends DiPerna-Lions’ renormalization techniques to nonlinear PDE.

Francesco Grotto (Scuola Normale Superiore)

CLT for Point Vortices and 2D Euler Invariant Measures

Abstract: We draw parallels between Gibbsian ensembles of Euler Point Vortices and the Energy-Enstrophy Gaussian invariant measures for the 2D Euler equations. A fine control of partition functions of point vortices ensembles allows to show a Central Limit Theorem which sees such ensembles converge to the aforementioned Gaussian measures in a suitable scaling regime. Joint work with Marco Romito.

Martina Hofmanová (Universität Bielefeld)

Solvability and ill-posedness of the isentropic Euler system

Abstract: I will discuss several puzzling results related to solvability and ill-posedness of the isentropic Euler system. On the one hand, the method of convex integration can be used to construct infinitely many wild solutions as well as rather surprising approximation results. On the other hand, ideas from Markov selections and a new notion of dissipative solution allow to select a semiflow of physically reasonable solutions and to exclude oscillation defects in certain cases.

Dejun Luo (Chinese Academy of Sciences)

High mode transport noise improves vorticity blow-up control in 3D Navier-Stokes equations

Abstract: We show that a suitable multiplicative noise of transport type has a regularizing effect for the vorticity form of 3D Navier-Stokes equations on the torus. It is proven that, given a time interval $[0,T]$, stochastic transport noise provides a bound on vorticity on such interval which gives well
posedness, with high probability. The result holds for sufficiently large noise intensity and sufficiently high spectrum of the noise. This is a joint work with Franco Flandoli.

Vincent R Martinez (City University of New York)

Controllability and Unique Ergodicity for the stochastic damped-driven KdV equation

Abstract: This talk will discuss the issue of establishing the uniqueness of invariant measures for the damped-driven stochastic KdV equation by adapting a classical control argument. Due to the structure of the equation and the non-regularizing dissipation, the usual asymptotic coupling argument encounters a criticality that cannot be breached. It will be seen, however, that the strengthening of the Doob-Khasminskii theorem established by Hairer-Mattingly can be verified directly to ensure the uniqueness result. This is joint work with Nathan Glatt-Holtz (Tulane University) and Geordie Richards (Utah State University).

Klas Modin (Chalmers University of Technology & University of Gothenburg)

Long-time behaviour of 2D spherical ideal hydrodynamics

Abstract: Using quantization theory we develop a new discretization scheme for the 2D vorticity equation on a sphere. The scheme preserves all of the underlying geometric features: conservation of infinitely many Casimir functions and Lie-Poisson structure. We identify a new mechanism which connects the long-time behaviour with integrability of low-dimensional point vortex dynamics.

Francesco Morandin (Università degli Studi di Parma)

Turbulence, shell models and critical exponents for dissipation

Abstract: Shell models of turbulence are nonlinear dynamical systems inspired by fluid dynamics. They are idealized and simplified, but tailored to exhibit the same energy cascade behaviour of three dimensional Euler and Navier-Stokes equations. A typical feature of these models is in fact anomalous dissipation of energy, which in finite time “escapes” to infinity, yielding a blow-up and instantaneous loss in regularity. A dissipative term corresponding to viscosity can recover regularity, for some of these models, but in total generality one needs hyper-dissipation, with an exponent larger than the physical one. Recent results hint that in the more refined framework of tree (hierarchical) models the required exponent may be actually lower.

Torstein Nilssen (University of Agder)

Rough perturbations of the Navier-Stokes system

Abstract: In this talk I will discuss 2 different ways of perturbing the Navier-Stokes equation by rough paths and the corresponding physical relevance in terms of conserved quantities. The presentation will focus on how to define intrinsic notions of solutions of the equations and corresponding well-posedness results. Joint work with Martina Hofmanová and James-Michael Leahy.
Camilla Nobili (Universität Hamburg)

Rigorous bounds on scaling laws in fluid dynamics

Abstract: We are interested in thermal convection as described by the Rayleigh-Bénard convection model. In this model the Navier-Stokes equations for the (divergence-free) velocity $u$ with no-slip boundary conditions are coupled to an advection-diffusion equation for the temperature $T$ with inhomogeneous Dirichlet boundary conditions. The problem of understanding the (average) upward-heat-transport properties is of great interest for the applications and challenging for the rigorous analysis. We show how the PDE theory (in particular, regularity analysis) can contribute to the understanding of the scaling regimes for the heat transport. After reviewing the theory of Constantin & Doering '99 we will present some recent results and discuss new challenges.

Enrico Priola (Università di Pavia)

An optimal regularity result for Kolmogorov equations with applications to some singular SPDEs

Abstract: We consider infinite dimensional Kolmogorov equations in a separable Hilbert space $H$ having singular first order terms. We prove an optimal regularity result for solutions to such equations. This result allows to study semilinear SPDEs of the form $dX_t = AX_t dt + (-A)\gamma F(X_t) dt + dW_t$ driven by a cylindrical Wiener process $W = (W_t)$; here $A$ is a suitable self-adjoint operator on $H$.

Marco Romito (Università di Pisa)

Generalized point vortex models

Abstract: We consider a class of models that slightly generalize the 2D Euler equations and that admit a description in terms of points vortex models. The interactions among vortices is more singular than the Euler case, nevertheless we develop a statistical mechanics theory for point vortices. We present a variational approach to the mean field limit of point vortices and an analysis of fluctuations in the case of the torus. This is a work in collaboration with C. Geldhauser (Dresden).

Martin Saal (Technische Universität Darmstadt)

The stochastic bidomain problem

Abstract: We consider the bidomain problem with FitzHugh-Nagumo transport. The linear part of this system is described by an operator, which admits a bounded $H^\infty$-calculus. In a recent work by Hieber and Prüß this was used to prove the global existence of solutions for rather rough initial data by deducing energy estimates and applying the theory of critical spaces. We show how to combine their ideas with a result on stochastic maximal regularity to obtain a global solution to the bidomain problem with noise. This is joint work with Matthias Hieber (TU Darmstadt) and Amru Hussein (TU Kaiserslautern).
Alexander Schmeding (University of Bergen)

A geometric view on stochastic Euler equations

Abstract: We consider a stochastic version of Euler equations. Due to a trick devised by V. Arnold in the deterministic setting, one can rewrite certain stochastic PDEs on a compact manifold as a stochastic differential equation on an infinite-dimensional manifold. Applying the machinery developed by Ebin and Marsden (in the deterministic case), the existence and uniqueness of a strong solution in spaces of Sobolev mappings (of high enough regularity) can be established. Our approach combines techniques from stochastic analysis and infinite-dimensional geometry and provides a novel toolbox to establish local well-posedness of stochastic non-linear partial differential equations. This is joint work with M. Maurelli and K. Modin, cf. arXiv:1909.09982.