HIM HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop on

"Probabilistic and variational methods in kinetic theory"

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organized by

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Abstracts

Anton Arnold (TU Wien)

Short- and long-time behavior in (hypo)coercive ODE-systems and Fokker-Planck equations

Abstract: We are concerned with the large-time behavior and the computation of the L^2-propagator norm of Fokker-Planck equations with linear drift, i.e. $\partial_t f = div(D\nabla_x f + Cxf)$. First we realize that a coordinate transformation can normalize these equations such that the diffusion and drift matrices are linked as $D = C_s$, the symmetric part of C. The main result of this talk is the connection between normalized Fokker-Planck equations and their drift-ODE $\dot{x} = -Cx$: Their L^2-propagator norms actually coincide. This implies that optimal decay estimates on the drift-ODE (w.r.t. both the maximum exponential decay rate and the minimum multiplicative constant) carry over to sharp exponential decay estimates of the Fokker-Planck solution towards the steady state. This result is based on the fact that the evolution in each invariant spectral subspace can be represented as an explicitly given, tensored version of the corresponding drift-ODE. In fact, the Fokker-Planck eqaution can even be considered as the second quatization of $\dot{x} = -Cx$.

References:
* A. Arnold, C. Schmeiser, B. Signorello. Sharp decay estimates and L^2-propagator norm for Fokker-Planck equations with linear drift, preprint 2019.
* A. Arnold, J. Erb. Sharp entropy decay for hypocoercive and non-symmetric Fokker-Planck equations with linear drift, arXiv 2014.
* F. Achleitner, A. Arnold and D. Stürzer: Large-time behavior in non-symmetric Fokker-Planck equations; Rivista di Matematica della Università di Parma 6 (2015) 1-68.

Giada Basile (University of Rome La Sapienza)

A gradient flow approach to kinetic equations

Abstract: I will present some results obtained together with D. Benedetto and L. Bertini on a gradient flow formulation of linear kinetic equations, in terms of an entropy dissipation inequality. The setting includes the current as a dynamical variable. I will show the relation between this formulation and the large deviation principle for continuous time Markov chains and I will discuss how this approach could be extended to the non linear case.

Martin Burger (FAU Erlangen-Nürnberg)

Propagation of gradient flow structures from microscopic to macroscopic models

Abstract: In this talk we will discuss the propagation of gradient flow structures from microscopic models in statistical mechanics such as overdamped particle dynamics or interacting particle systems on lattices to macroscopic partial differential equations. The key insight is that microscopic models can be formulated as linear Markov chains in high-dimensional spaces, e.g. via Liouville equations, for which recent work by Maas, Mielke and others has provided a rather complete picture. The propagation to macroscopic models is then carried out - at least formally - by constructing a metric structure on an associated infinite hierarchy of equations, resembling the BBGKY hierarchy in kinetic theory, and studying mean-field or other limits in this setup.

Li Chen (Universität Mannheim)

Mean field limit of many particle system with non Lipschitz force

Abstract: We apply a probabilistic method to derive the mean field limit for an interacting particle model in two dimensions. The model under investigation contains the velocity alignment effect in pedestrian flow and convey band problems. For stochastic initial data, we prove the convergence in measure of the N-particle system to the solution of the Vlasov equation with properly chosen cut-off. Results on the propagation of chaos will be also deduced. Furthermore, the existence of weak solution of the Vlasov equation with velocity alignment effect are investigated, which is needed in obtaining the mean field limit.

Matias Delgadino (Imperial College London)

Mean field limit by Gamma convergence

Abstract: In this work we give a proof of the mean-field limit for λ-convex potentials using a purely variational viewpoint. We take advantage that all evolutions of the involved quantities can be written as gradient flows of functionals at different levels: in the set of symmetric probability measures on N variables and in the set of probability measures on probability measures. This basic fact allows us to rely on Γ-convergence tools for gradient flows to finish the proof by identifying the limits of the different terms in the Evolutionary Variational Inequalities (EVIs) associated to each gradient flow. The λ-convexity of the potentials is crucial to identify uniquely the limits and in order to derive the EVIs at each description level of the interacting particle system. This is joint work with J.A. Carrillo and G. Pavliotis.

Laurent Desvillettes (Université Paris Diderot)

A link between cross diffusion and predator prey systems

Abstract: The so called functional responses appearing in predator prey systems can be explained thanks to the use of fast reaction limits or quasi steady state approximations at the level of ODEs. When those limits are considered for reaction-diffusion equations instead of ODEs, they lead to cross diffusion models differing from the reaction-diffusion systems involving the classical functional responses. We explain the mathematical properties of those cross diffusion models at the level of existence/smoothness and also at the level of large time behavior and instabilities, and compare them to the corresponding reaction-diffusion systems.

Helge Dietert (IMJ-PRG)

About the entropic structure of detailed balanced multi-species cross-diffusion equations

Abstract: This talk links at the formal level the entropy structure of a multi-species cross-diffusion system of Shigesada-Kawasaki-Teramoto (SKT) type satisfying the detailed balance condition with the entropy structure of a reversible microscopic many-particle Markov process on a discretised space. The link is established by first performing a mean-field limit to a master equation over discretised space. Then the spatial discretisation limit is performed in a completely rigorous way, which by itself provides a novel strategy for proving global existence of weak solutions to a class of cross-diffusion systems. This is a joint work with Esther S. Daus and Laurent Desvillettes.

Andreas Eberle (Universität Bonn)

Couplings and convergence to equilibrium for Langevin dynamics and Hamiltonian Monte Carlo methods

Abstract: Coupling methods provide a powerful approach to quantify convergence to equilibrium of Markov processes in appropriately chosen Wasserstein distances. This talk will give an overview on two closely relately, recently developed coupling approaches for kinetic Langevin equations and for Hamiltonian Monte Carlo methods (HMC). The approaches do not require convexity, and they yield bounds of kinetic order if the damping coefficient in the Langevin equation or the integration length in HMC are adjusted appropriately. The talk is based on joint work with Arnaud Guillin, Raphael Zimmer and Nawaf Bou-Rabee.

Amit Einav (Karl-Franzens-Universität Graz)

Weak Poincaré inequalities in the absence of spectral gaps

Abstract: Weak Poincaré inequalities in the absence of spectral gaps Abstract: Poincaré inequality, which is probably best known for its applications in PDEs and calculus of variation, is one of the simplest examples to an inequality that lies in the crossroads of Analysis, Probability and Semigroup/Spectral theory. It can be understood as the functional inequality that arises from attempting to understand convergence of the so-called heat flow to its equilibrium state. This approach can be generalised to the setting of Markov semigroups, with a non-positive generator that posses a spectral gap. A natural question that one can consider is: What happens if the generator doesn't have a spectral gap? Can we still deduce a rate of convergence from a functional setting? In this talk we will discuss a new approach to this question and see how an understanding of the way the spectrum of the generator behaves near the origin, in the form of a density of states estimate, can lead to weak Poincare type inequalities, from which a quantitative estimation of convergence can be obtained. This talk is based on a joint work with Jonathan Ben Artzi.

Matthias Erbar (Universität Bonn)

A gradient flow approach to the Boltzmann equation

Abstract: In this talk I will present an interpretation of the spatially homogeneous Boltzmann equation as the gradient flow of the entropy. This gradient flow structure relies on a geometry on the space of probability measures that takes the collision process between particles into account. This point of view leads to a new approach to proving propagation of chaos for Kac's random walk and its convergence to the Boltzmann equation.

Antonio Esposito (University of L'Aquila)

Nonlinear degenerate cross-diffusion systems with nonlocal interaction

Abstract: I will discuss a joint work with M. Di Francesco (University of L'Aquila) and S. Fagioli (University of L'Aquila). We investigate a class of systems of partial differential equations with nonlinear cross-diffusion and nonlocal interactions, which are of interest in several contexts in social sciences, finance, biology, and real world applications. Assuming a uniform 'coerciveness' assumption on the diffusion part, which allows to consider a large class of systems with degenerate cross-diffusion (i.e. of porous medium type) and relaxes sets of assumptions previously considered in the literature, we prove global-in-time existence of weak solutions by means of a semi-implicit version of the Jordan-Kinderlehrer-Otto scheme. Our approach allows to consider nonlocal interaction terms not necessarily yielding a formal gradient flow structure.

Jo Evans (Université Paris Dauphine)

Using Harris's theorem to prove hypocoercivity for linear kinetic equations with jumps

Abstract: I will explain how Harris's theorem which is a classical theorem from Markov processes can be applied to spatially inhomogeneous kinetic equations to show convergence to equilibrium. Convergence to equilibrium for such equations with quantitative rates are usually shown using techniques known as hypocoercivity. I will compare the use of Harris's theorem to the classical techniques. We will work in the context of the linear relaxation Boltzmann equation.

Simone Fagioli (University of L'Aquila)

Solutions to aggregation–diffusion equations with nonlinear mobility constructed via a deterministic particle approximation

Abstract: We construct a deterministic, Lagrangian many-particle approximation to a class of nonlocal transport PDEs with nonlinear mobility coupled with nonlinear diffusion, arising in many contexts in biology and social sciences. The approximating particle system is a nonlocal version of the Follow-the-Leader scheme. In the purely aggregative case, we rigorously prove that a suitable discrete piecewise density reconstructed from the particle scheme converges strongly towards the unique entropy solution to the target PDE as the number of particles tends to infinity. The proof is based on uniform BV estimates on the approximating sequence and on the verification of an approximated version of the entropy condition for large number of particles. In presence of diffusion, we prove convergence of the aforementioned scheme to weak solutions of the aggregation–diffusion PDEs. The main novelties concern the presence of a nonlinear mobility term and the non-strict monotonicity of the diffusion function. As a consequence, our result applies also to strongly degenerate diffusion equations. This is a joint work with M.Di Francesco and E. Radici (University of L'Aquila)

Joaquin Fontbona (University of Chile)

Quantitative uniform propagation of chaos for Maxwell molecules

Abstract: We prove propagation of chaos at explicit, mild polynomial rates in Wasserstein distance W_2, for Kac's N-particle system associated with the spatially homogeneous Boltzmann equation for Maxwell molecules (with and without cutoff). Our approach is based on novel, optimal transportbased probabilistic coupling techniques, developed in order to deal with (genuine) binary-jump interactions, and on a recent stabilization result for the particle system (obtained by M.Rousset). In particular, under suitable moments assumptions on the initial distribution, we establish a uniform-in-time estimate of order almost $N^{-1/3}$ for W_2^2. Joint work with Roberto Cortez.

Giambattista Giacomin (Université Paris Diderot)

Interacting diffusions on random graphs and PDE limits

Abstract: Starting from the classical problem of mean field (i.e., complete graph) interacting diffusions, whose empirical measure obeys a McKean-Vlasov (or Fokker-Planck) PDE in the limit of infinitely many interacting units, we investigate the stability of this result when the complete graph is replaced by a random graph with a suitable uniform degree property. The talk will be a review of the results and of (some of) the methods available for this problem. The emphasis will be on explaining why, in spite of the fact that these results may look rather general and satisfactory, some crucial issues are widely open.

Rishabh Gvalani (Imperial College London)

A mountain pass theorem in the space of probability measures and applications

Abstract: We prove a version of the mountain pass theorem for lower semicontinuous and λ-geodesically convex functionals on the space of probability measures $P(M)$ equipped with the W_2 Wasserstein metric, where M is a compact Riemannian manifold or R^d. As an application of this result, we show that the empirical process associated to a system of weakly interacting diffusion processes exhibits a form of noise-induced metastability. The result is based on an analysis of the associated McKean–Vlasov free energy, which for suitable attractive interaction potentials has at least two distinct global minima at the critical parameter value $\beta = \beta_c$. Joint work with André Schlichting.

Franca Hoffmann (California Institute of Technology)

Reversed Hardy-Littlewood-Sobolev Inequalities

Abstract: We will introduce a new family of reverse Hardy-Littlewood-Sobolev inequalities which involve a power law kernel with positive exponent. We investigate the range of the admissible parameters and the properties of the optimal functions. A striking open question is the possibility of concentration which is analyzed and related with free energy functionals and nonlinear diffusion equations involving mean field drifts. This is joint work with José A. Carrillo, Matias G. Delgadino, Jean Dolbeault, and Rupert L. Frank.

Ansgar Jüngel (TU Wien)

Cross-diffusion models for multispecies systems in biology: modeling, entropies, stochastics

Abstract: The description of the dynamics of individuals for multiple species (e.g. populations, biological cells, ions) on the macroscopic level often leads to strongly coupled parabolic systems. The diffusion matrix of these cross-diffusion systems is typically neither symmetric nor positive definite, which complicates the analysis. The idea is to reveal a gradient-flow or entropy structure, which allows for gradient estimates and in certain applications for lower and upper bounds without applying a maximum principle. In this talk, the entropy structure is explained and the global existence of weak solutions is proved. Furthermore, a population cross-diffusion system with stochastic forcing (multiplicative noise) is discussed and the global existence of martingale solutions is shown.

Shi Jin (Shanghai Jiao Tong University)

Random batch methods for interacting particle systems

Abstract: We develop random batch methods for interacting particle systems with large number of particles. These methods use small but random batches for particle interactions, thus the computational cost is reduced from $O(N^2)$ per time step to $O(N)$, for a system with N particles with binary interactions. On one hand, these methods are efficient Asymptotic-Preserving schemes for the underlying particle systems, allowing N-independent time steps and also capture, in the N $\to \infty$ limit, the solution of the mean field limit which are nonlinear Fokker-Planck equations; on the other hand, the stochastic processes generated by the algorithms can also be regarded as new models for the underlying problems. For one of the methods, we give a particle number independent error estimate under some special interactions. Then, we apply these methods to some representative problems in mathematics, physics, social and data sciences, including the Dyson Brownian motion from random matrix theory, Thomson's problem, distribution of wealth, opinion dynamics and clustering. Numerical results show that the methods can capture both the transient solutions and the global equilibrium in these problems. This is a joint work with Lei Li (Shanghai Jiao Tong University) and Jian-Guo Liu (Duke University)

Jean-Christophe Mourrat (ENS Ecole Normale Superieure)

Energy methods for the kinetic Fokker-Planck equation

Abstract: The kinetic Fokker-Planck equation is the PDE associated with the dynamics of a particle subject to a white noise forcing in its velocity. The goal of my talk will be to recast several results such as well-posedness, regularity and exponential decay to equilibrium over long times using ideas similar to classical energy methods for elliptic equations.

Michela Ottobre (Heriot Watt University)

A one-dimensional Vicsek-type model for self-propelled diffusions

Abstract: The study of interacting particle systems of self-propelled particles has attracted attention for now a few decades and has posed interesting challenges to standard paradigms in statistical mechanics. Such models are at the root of many biological phenomena, such as bacterial migration, flocking of birds etc. In this talk we will consider a new continuum model of Viksek-type. Such a model

is constituted by a non-linear PDE which is i) not in gradient form and ii) it is non-uniformly elliptic (but hypoelliptic instead). Moreover, as typical in this framework, the dynamics exhibits multiple equilibria (stationary states). This is a joint work with P.Butta (La Sapienza, Rome), F. Flandoli (Scuola Normale, Pisa) and B. Zegarlinski (Imperial College).

Francesco Patacchini (Carnegie Mellon University)

The interaction equation near attracting manifolds

Abstract: We study the existence and uniqueness of solutions to the interaction equation on manifolds, where the velocity is the projection of the Euclidean velocity onto the tangent space of the manifold. For the uniqueness, a stability estimate can be obtained under the assumption that the manifold be of positive reach. For the existence, we approximate the interaction energy on the manifold by the classical full-space interaction energy to which we add a confinement part which is proportional to the distance to the manifold and which blows up as a parameter epsilon tends to 0. This ensures that any mass near the manifold gets attracted towards it as epsilon vanishes. We use the Sandier-Serfaty approach to show that, in the limit as epsilon goes to 0, the gradient flow in the full space converges to the gradient flow on the manifold. This is ongoing work with Dejan Slepcev.

Mario Pulvirenti (Universita di Roma La Sapienza)

A local mean-field stochastic particle system for the Bolzmann equation

Abstract: -

Angela Stevens (Universität Münster)

Alignment and Wavenumber Selection

Abstract: A kinetic model for the alignment of bacteria is discussed. Conditions for the selection of orientations and mass distributions for long times are given in the limit of small deviations of interactions between the bacteria. In contrast to the related kinetic equation with deterministic turning, here a selection of mass occurs due to noise. After alignment, the bacteria show one-dimensional run-and-tumble dynamics. Nonlinearities in tumbling rates induce the existence of a plethora of traveling- and standing-wave patterns, as well as a subtle selection mechanism for the wavenumbers of spatio-temporally periodic waves. Joint works with: Ivano Primi, Arnd Scheel, and Juan J. L. Velazquez.

Bao Q. Tang (University of Graz)

Indirect diffusion effect and convergence to equilibrium

Abstract: We present in this talk a phenomenon called indirect diffusion effect in studying convergence to equilibrium by entropy method for reaction-diffusion systems, typically arising from chemical reactions. It may happen in such systems that some species do not diffuse, which causes a major difficulty as the Fischer information is missing, and consequently one does not have the logarithmic Sobolev inequality for those species. The indirect diffusion effect asserts that the combination of diffusion from diffusing species and reversible reactions will lead to some diffusion effect on non-diffusing species, which in turn drives the whole system towards the chemical equilibrium.

Hagop Tossounian (Universidad de Chile)

On Kac's model, ideal Thermostats, and finite Reservoirs

Abstract: In 1956, Mark Kac introduced a stochastic model to derive a Boltzmann-like equation. Like the space-homogeneous Boltzmann's equation, Kac's equation is ergodic with centered Gaussians as the unique equilibrium state. In this talk, I will introduce Kac's model, the thermostat used in [1] to guarantee exponentially fast convergence to equilibrium, and sketch the result in [2] how this (infinite) thermostat can be approximated by a finite but large reservoir.
[1] Bonetto, F., Loss, M.,Vaidyanathan, R.: J. Stat. Phys. 156(4), 647– 667 (2014)
[2] Bonetto F., Loss, M. Tossounian, H. Vaidyanathan, R. Comm. Math. Phys. 351 (2017)

Oliver Tse (Eindhoven University of Technology)

Equilibration in Wasserstein distance of partially damped Euler equations

Abstract: We discuss ideas and tools to construct Lyapunov functionals on the space of probability measures to investigate convergence to global equilibrium of partially damped Euler equations under the influence of external and interaction potential forces with respect to the 2-Wasserstein distance.

Boguslaw Zegarlinski (Imperial College London)

Coercive inequalities for Markov generators on nilpotent Lie groups

Abstract: I will review and present some new results on construction, long time behaviour and coercive inequalities for Markov semigroups on nilpotent Lie groups.
