HAUSDORFF RESEARCH INSTITUTE
FOR MATHEMATICS

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Follow-Up-Workshop on

"Von Neumann Algebras"

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organized by

Dietmar Bisch (Nashville), Sorin Popa and Dimitri Shlyakhtenko (Los Angeles)

Abstracts

Arnaud Brothier (University of New South Wales)

Forest-skein groups

Abstract: Vaughan Jones found unexpected connections between subfactor theory and Richard Thompson's group while attempting to construct conformal field theories (CFT). This led to numerous fruitful applications and among others provided a novel way to construct group actions using categories. I am initiating a program strengthening Jones' visionary work where the Thompson group is replaced by a family of groups that I name "forest-skein groups". These groups are interesting on their own, satisfying exceptional properties and having powerful extra-structures for studying them. They are built via planar diagrams mod out by certain skein relations just like planar algebras.

I will briefly tell the story of Jones's discovery and then present forest-skein groups. Explicit examples will be given as well as concrete applications in group theory.

Rolando de Santiago (Purdue University)

Bounds on the quantum chromatic number of lexicographical products of quantum graphs

Abstract: We provide a brief introduction to quantum graphs and the quantum chromatic number of graphs. We define the lexicographic product of quantum graphs and investigate bounds on the quantum chromatic number of the resulting graph products. Additionally, we discuss our work on developing bounds for the lexicographic product of quantum graphs which are analogous to those in the classical case using fractional quantum chromatic numbers. This is joint work with A. Meenakshi McNamara.

Daniel Drimbe (KU Leuven)

Embedding universality for II_1 factors with property (T)

Abstract: In this talk we discuss a joint work with Ionut Chifan and Adrian Ioana in which we prove that every separable tracial von Neumann algebra embeds into a II_1 factor with property (T) which can be taken to have trivial outer automorphism and fundamental groups. We also establish an

analogous result for the trivial extension over a non-atomic probability space of every countable p.m.p. equivalence relation.

David Evans (Cardiff University)

Higher equivariant twists, K-theory and fusion rings

Abstract: This talks is part of a programme to understand conformal field theory through K-theory, in particular twisted equivariant K-theory.

Freed-Hopkins-Teleman realized the Verlinde fusion ring of positive energy representations of loop groups through twisted equivariant K-theory of the section algebra of equivariant bundles of compact operators.

In work with Ulrich Pennig, we employ exponential functors on the category of finite-dimensional complex inner product spaces and isomorphisms to provide an equivariant higher (ie. non-classical) twist of K-theory employing the section algebra of a locally trivial bundle with stabilised strongly self-absorbing fibres.

Samuel Evington (University of Muenster)

Actions of unitary fusion categories on the Jiang Su algebra

Abstract: Any unitary fusion category acts on the hyperfinite II 1 factor, by results of Popa and Ocneanu. In there C-setting, there are K-theoretic obstructions, developed by Kajiwara and Watatani, that place restrictions on the dimensions of objects in the fusion category that can act on a given C algebra.
In this, talk I will discuss an additional obstruction to actions of the fusion categories Hilb G omega coming from algebraic K 1. In particular, it follows that there are no fully faithful actions of Hilb G omega on the Jiang Su algebra. This talk is based on joint work with Sergio Giron Pacheco.

Pinhas Grossman (University of New South Wales)

Graded extensions of generalized Haagerup categories

Abstract: We describe a construction of extensions of Haagerup-Izumi categories by outer automorphisms, which leads to a number of new examples of fusion categories. There is an obstruction theory for graded extensions of fusion categories due to Etingof, Nikshych, and Ostrik. However, computing the obstructions can be difficult in concrete examples. We get around this by directly constructing the extensions using operator algebras (in particular free products of Cuntz algebras and free group C*-algebras). This is joint work with Masaki Izumi and Noah Snyder.

Ben Hayes (University of Virginia)

Consequences of the recent resolution of the Peterson-Thom conjecture

Abstract: The Peterson-Thom conjecture states that every diffuse, amenable subalgebra of a free group factor is contained in a unique maximal amenable subalgebra. Previous work of mine reduced this conjecture to a generalization of the Haagerup-Thorbjornsen theorem on strong convergence of random matrices. This problem in random matirces was recently solved Belinschi-Capitaine. I will

explore several applications of the combination of their results with mine. These include a large generalization of the celebrated strong solidity theorem of Ozawa-Popa, verification of the coarse conjecture independently due to myself and Popa, general results on amenable and Gamma absorption for maximal amenable subalgebras of free group factors, general results on left/right measure for maximal abelian subalgebras of free group factors which are maximal injective, along with other results.

Cyril Houdayer (Université Paris-Saclay)

Pointwise inner automorphisms of almost periodic factors

Abstract: In this talk, I will show that a large class of almost periodic type III_1 factors M, including all McDuff factors that tensorially absorb R_∞ and all free Araki-Woods factors, satisfy Haagerup-Størmer's conjecture (1988): any pointwise inner automorphism of M is the composition of an inner and a modular automorphism. In particular, this provides the first examples of nonamenable type III_1 factors satisfying Haagerup-Størmer's conjecture. This is joint work with Yusuke Isono.

David Jekel (UCSD)

Connections between free probability and model theory

Abstract: Motivated by recent developments in model theory of operator algebras, we consider free probabilistic questions for model-theoretic types rather than non-commutative laws. In particular, we study notions of free entropy and 1-bounded entropy for types. The model-theoretic type of $(X_1, \ldots, X_d)$ in a tracial W*-algebra $\mathcal{M}$ encodes more information than only the $*$-moments, namely it considers the values of formulas obtained by applying functions to traces of non-commutative polynomials in X and other variables Y and then taking suprema and infima (i.e. quantifiers in continuous model theory) over some of the auxiliary variables. Because the theory of diffuse classical probability spaces admits quantifier elimination, there is no distinction between the law and the type, but in the non-commutative setting the type may be a better analog for a probability distribution. We adapt the notions of free entropy and 1-bounded entropy by looking at spaces of matrix approximations for a type rather than a non-commutative law, and give several applications to matrix ultraproducts; in particular, we show that (in contrast to the ultrapower of $\mathcal{R}$) a matrix ultraproduct is not strongly 1-bounded.

Corey Jones (North Carolina State University)

K-theoretic classification of inductive limit actions of fusion categories on AF-algebras

Abstract: Using ideas from the theory of subfactors, we introduce a K-theoretic invariant for actions of fusion categories on C*-algebras. For actions which are inductive limits of finite dimensional actions, we show this is a complete invariant, extending Elliott's theorem to the quantum symmetry-enriched case. We will discuss implications for the classification of both finite group actions on AF-algebras and inclusions of AF-algebras. Based on joint work with Quan Chen and Roberto Hernández Palomares.

Srivatsav Kunnawalkam Elayavalli (IPAM, UCLA)

On proper proximality for groups and von Neumann algebras

Abstract: In this talk I will motivate and define the notion of proper proximality for groups (due to Boutonnet Ioana and Peterson) and then for von Neumann algebras. I will describe several examples and some permanence properties in either instance. I will then describe several applications of this property to questions about II_1 factors. This is joint work with Changying Ding and Jesse Peterson.

Alexey Kuzmin (University of Gothenburg)

New models of old operator algebras

Abstract: I will discuss how well-known operator algebras: Cuntz algebras (O_n, O_∞), noncommutative torus, canonical commutation relations, quantum groups, appear in a disguise of boundary Wick algebras.

Brent Nelson (Michigan State University)

Free Stein dimension

Abstract: Let (M, τ) be a tracial von Neumann algebra and A a finitely generated *-subalgebra of M. The free Stein dimension of (A, τ) is the von Neumann dimension of a module determined by certain closable derivations on A. In particular, it is a free probabilistic invariant associated to the non-commutative distribution of generators for A with respect to τ, and it can be easily computed for many examples. In this talk, I will provide an introduction to free Stein dimension and show how its quantitative behavior can reveal structural properties of the von Neuman algebra A''. This is based on joint work with Ian Charlesworth.

Sergey Neshveyev (Oslo University)

Furstenberg-Hamana boundary of a C*-tensor category

Abstract: Motivated by recent interest in developing a Furstenberg type boundary theory in various noncommutative contexts, I will explain how one can associate a Furstenberg-Hamana boundary to every rigid C*-tensor category with simple unit. For a large class of tensor categories this boundary turns out to coincide with a Poisson type boundary arising from an earlier joint work with Makoto Yamashita. For the representation categories of the q-deformations of compact Lie groups this can be viewed as a categorical (or quantum group theoretic) analogue of the classical computation of the Furstenberg boundaries of complex semisimple Lie groups by Furstenberg and Moore. (Joint work with Erik Habbestad and Lucas Hataishi.)

David Penneys (The Ohio State University)

A categorical Connes' $\chi(M)$

Abstract: Popa introduced the tensor category $\tilde{\chi}(M)$ of approximately inner, centrally trivial bimodules of a II_1 factor M, generalizing Connes' $\chi(M)$. We extend Popa's notions to define the W*-tensor category $\mathrm{End}_{loc}(\mathcal{C})$ of local endofunctors on a W*-category $\mathcal{C}$. We construct a unitary braiding on

$\mathrm{End}_{loc}(\mathcal{C})$, giving a new construction of a braided tensor category associated to an arbitrary W*-category. For the W*-category of finite modules over a II_1 factor, this yields a unitary braiding on Popa's $\tilde{\chi}(M)$, which extends Jones' κ invariant for $\chi(M)$.

Given a finite depth inclusion $M_0 \subseteq M_1$ of non-Gamma II_1 factors, we show that the braided unitary tensor category $\tilde{\chi}(M_\infty)$ is equivalent to the Drinfeld center of the standard invariant, where M_∞ is the inductive limit of the associated Jones tower. This implies that for any pair of finite depth non-Gamma subfactors $N_0 \subseteq N_1$ and $M_0 \subseteq M_1$, if the standard invariants are not Morita equivalent, then the inductive limit factors N_∞ and M_∞ are not isomorphic.

This is joint work with Corey Jones and Quan Chen.

Thomas Sinclair (Purdue University)

A notion of index for finite-dimensional operator systems

Abstract: Inspired by a well-known characterization of the Jones index of an inclusion of II_1 factors due to Pimsner and Popa, we define an index-type invariant for inclusions of finite-dimensional operator systems. We will compute examples of this invariant, and explain how it generalizes the Lovasz theta invariant to general matricial systems in a manner that is closely related to the quantum Lovasz theta invariant defined by Duan, Severini, and Winter.

Noah Snyder (Indiana University)

An algebraic version of the small index subfactor classification

Abstract: The standard invariant of a subfactor $N \subset M$ can be thought of as a pair $(\mathcal{C}, A)$ where $\mathcal{C}$ is the C*-tensor category of N-N bimodules, and A is a C*-algebra object (aka Q-system) representing M. The index is then the dimension of A. Classification results for small index subfactors proceed by first classifying all subfactor standard invariants, and then turning to more difficult analytic questions about understanding how this invariant can be realized as acting on a particular factor M. From the representation theory point of view it's natural to consider pairs $(\mathcal{C}, A)$ where $\mathcal{C}$ is a semisimple tensor category without any C*-structure and A is a semisimple algebra object. There is an immediate problem in asking the analogue of the small index subfactor classification, because there are several possible definitions of dimension and $\dim A$ may not be positive! To this end we consider semisimple tensor categories $\mathcal{C}$ together with a fixed *positive* dimension function which assigns a positive number $d(X)$ to every object X of $\mathcal{C}$ and satisfies $d(X \oplus Y) = d(X) + d(Y)$ and $d(X \otimes Y) = d(X)d(Y)$. Our goal then is to classify triples $(\mathcal{C}, A, \mathrm{d})$ where $d(A)$ is small. Here our main result is that any such "small index algebraic subfactor" of index below $3 + \sqrt{3}$ is Galois conjugate to a subfactor standard invariant. The key step is a novel triple point obstruction. This is joint work with Penneys and builds on joint work with Penneys and Peters.

Roland Speicher (Saarland University)

Universality of free random variables: atoms for non-commutative rational functions

Abstract: Consider a tuple of normal operators in a tracial operator algebra setting with prescribed sizes of the eigenspaces for each of the operators. We address the question what one can say about the sizes of the eigenspaces for any non-commutative polynomial in those operators? We show that for each polynomial there are unavoidable eigenspaces. We will describe this minimal situation both in algebraic terms - where it is given by realizations via matrices over the free skew field and via rank

calculations - and in analytic terms - where it is given by freely independent random variables with prescribed atoms in their distributions. The fact that the latter situation corresponds to this minimal situation allows us to draw many new conclusions about atoms in polynomials of free variables.

The talk is based on arXiv:2107.11507, which is joint work with O. Arizmendi, G. Cebron, and S. Yin.

Hui Tan (UCSD)

Spectral gap characterizations of property (T) for II_1 factors

Abstract: I will discuss characterizations of property (T) for II_1 factors by weak spectral gap in inclusions into tracial von Neumann algebras. I will explain how this is related to the non-weakly-mixing property of the bimodules containing almost central vectors, where we obtain a II_1 factor version of a characterization of property (T) of Bekka and Valette.

Andreas Thom (TU Dresden)

Maximal discrete subgroups in unitary groups of operator algebras

Abstract: We show that if a group G is mixed-identity-free, then the projective unitary group of its group von Neumann algebra contains a maximal discrete subgroup containing G. The proofs are elementary and make use of free probability theory. In addition, we clarify the situation for C*-algebras. Similar results for full groups of p.m.p. equivalence relations are also discussed. This is joint work with Vadim Alekseev and Alessandro Carderi.

Stefaan Vaes (KU Leuven)

Nonsingular Bernoulli actions: a survey

Abstract: Given a countable group G and a base space X_0, the Bernoulli action is the translation action of G on the product space X_0^G, equipped with a product of probability measures μ_g on X_0. I will present a survey of recent results on the ergodicity and Krieger type of such nonsingular Bernoulli actions. I will in particular present a joint work with Tey Berendschot providing a complete answer when $G = \mathbb{Z}$ is the group of integers. It turns out that most, but not all injective factors can be written as the crossed product by a nonsingular Bernoulli action of $\mathbb{Z}$, since we prove that the associated Krieger flow must satisfy a divisibility property.

Hans Wenzl (UCSD)

Deformations of subgroups for $SU(N)_k$

Abstract: We study the question whether we can obtain from a subgroup $H \subset SU(N)$ an analog in the corresponding fusion tensor category $SU(N)_k$. We have shown that this is possible for $H = SO(N)$ and also, for N even, for $H = Sp(N)$. We moreover also expect such analogs to exist for $H = Sp(N-1)$ if N is odd. This leads to more examples of subfactors. Moreover, by recent abstract classification results by Edie-Michell, these are essentially all subgroups for which this is possible. In particular, our examples give explicit descriptions of all non-exceptional module categories of $SU(N)_k$.

Stuart White (Oxford University)

Tracially complete C*-algebras

Abstract: Over the last decade a major theme has been the application of von Neumann methods in the structure and classification of stably finite C*-algebras, often with the idea of explicitly lifting results back from the von Neumann level to the C*-algebra setting. This works most crisply for C*-algebras with a unique trace, where one works with the associated GNS representation. In this talk, I'll introduce the framework of tracially complete C*-algebras as a general bridge between the W* and C*-theory, and discuss structure and classification results for these new type of operator algebras. This is joint work with Carrion, Castillejos, Evington, Gabe, Schafhauser and Tikuisis.
