

INTRODUCTION TO SHIMURA VARIETIES

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1. COURSE OUTLINE

Jan. 13th-17th 2014, H.I.M., Bonn (10h-12h MWF, 14h-16h T-Th).

- (1) Modular curves
- (2) Siegel varieties
- (3) Shimura varieties of PEL type
- (4) Selected topics, including Shimura varieties of Hodge type
- (5) Towards a p -adic Kudla program

2. BRIEF DESCRIPTION

2.1. Modular curves. This is the starting point of the theory. We will recall basics about elliptic curves and their moduli spaces. We will see how modular curves enter the statement of a geometric Jacquet-Langlands correspondence, alongside with p -adically uniformed Shimura curves. We will mention recent developments: modular curves of infinite level.

2.2. Siegel varieties. After modular curves, the simplest Shimura varieties to define are the moduli spaces of principally polarized abelian varieties with level structures. We will define those Siegel varieties in detail, and include some examples in genus two: the Siegel threefolds.

2.3. Shimura varieties of PEL type. Shimura varieties that arise as moduli spaces of abelian varieties with additional structures such as endomorphisms are still the most commonly encountered in the literature. For example, this class includes Hilbert modular varieties and many unitary Shimura varieties. We will describe the stratifications of their special fibers defined via Barsotti-Tate groups, and illustrate this with generalized Hasse invariants.

2.4. Selected Topics (depending on the audience's interests).

2.4.1. Shimura varieties of Hodge type and the general formalism. I will introduce succinctly the general formalism of Shimura datum, define Shimura varieties of Hodge type and revisit previous examples in this new language. Research papers on Shimura varieties are now being written for general Hodge type Shimura varieties rather than PEL Shimura varieties of type A and C . In particular, varieties associated to GSpin and orthogonal Shimura varieties are now more routinely included.

2.4.2. Compactifications as "black boxes". I will try to sketch from a user's perspective the toroidal and minimal compactifications available to us for Shimura varieties of Hodge type at places of good reduction. Example: GSp_{2g} .

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2.4.3. *Deformation theory.* Examples: Siegel varieties, Hilbert modular varieties and unitary Shimura varieties.

2.5. **Towards a p -adic Kudla program.** I will try to explain the general picture of what I envision as an emerging (and quite speculative) p -adic Kudla program, including theoretical evidence related to $GL(2)$ and liftings thereof.

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