Title: Distinguishing Groups and the Group Isomorphism problem
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Abstract. Attacks on the Group Isomorphism Problem have diversified in the last decade with approaches ranging from Erdős-Rényi type models [8], to hyper-graph isomorphism techniques [1, 7], and several representation and Lie theory methods [3, 4, 10]. We will summarize necessary foundations and highlight important recent progress while concentrating on examples.

As our main illustration, we will use a method from the study of groups of finite Morley rank [2, 9]. We construct a family of non-isomorphic groups that share all the same proper subgroups, all the same proper quotient groups, the same character tables, and the same automorphism groups [11]. This settles a question of Gowers and Babai about the minimum requirements to decide isomorphism [6]. And yet, we also prove a polylogarithmic isomorphism test for this indistinguishable family of groups [11]. As these and other examples demonstrate we have a lot yet to learn about most groups.

References

[1] L. Babai, P. Codenotti, J. A. Grochow, Y. Qiao, *Code equivalence and group isomorphism*, Proc. 22 ACM-SIAM Symposium on Discrete Algorithms (2011) 1395–1408.
[2] A. Borovik, A. Nesin, *Groups of finite Morley rank* Oxford Logic Guides 26, The Clarendon Press, Oxford University Press, 1994.
[3] P. A. Brooksbank, J. Maglione, J. B. Wilson, *A fast isomorphism test for groups whose Lie algebra has genus 2*, J. Algebra 473 (2017) 545–590.
[4] P. A. Brooksbank, E. A. O'Brien, J. B. Wilson *Isomorphism testing of Graded algebras* arXiv:1708.08873.
[5] H. Dietrich, J. B. Wilson *Polynomial-time isomorphism testing of groups of most finite orders* arXiv:1806.08872.
[6] G. Glauberman, Ł. Grabowski, *Groups with identical k-profiles*, Theory Comput. 11 (2015) 395–401.
[7] J. A. Grochow, Y. Qiao, *Algorithms for group isomorphism via group extensions and cohomology*, SIAM J. Comput. 46 **4** (2017) 1153–1216.
[8] Y. Li, Y. Qiao, *Linear algebraic analogues of the graph isomorphism problem and the Erdős-Rényi model*, 58th Annual IEEE Symposium on Foundations of Computer Science—FOCS 2017, IEEE Computer Soc. (2017).
[9] A. G. Myasnikov, *The theory of models of bilinear mappings* Sibirsk. Mat. Zh. 31 **3** (1990) 94–108, 217.
[10] J. B. Wilson *On automorphisms of groups, rings, and algebras*, Comm. Alge. 45 (4) 2017, 1452–1478.
[11] J. B. Wilson *The threshold for subgroup profiles to agree is* $\log n - 2$ arXiv:1612.01444.