Predicative proof theory of PDL

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1 Abstract

Propositional dynamic logic **PDL** is presented in Schütte-style mode as one-sided semiformal tree-like sequent calculus $\mathrm{SEQ}_{\omega}^{\mathrm{PDL}}$ wiLth standard cut rule and the omega-rule with principal formulas $[P^{*}]A$. The omega-rule free derivations in $\mathrm{SEQ}_{\omega}^{\mathrm{PDL}}$ are finite (trees), while sequents deducible by these finite derivations are valid in **PDL**. Cut elimination theorem for $\mathrm{SEQ}_{\omega}^{\mathrm{PDL}}$ is provable in Peano Arithmetic (**PA**) extended by transfinite induction up to $\varphi_{\omega^{\omega}}(0) > \varepsilon_0$. Hence this predicative extension of **PA** proves that any given $[P^{*}]$-free sequent is valid in **PDL** iff it is deducible in $\mathrm{SEQ}_{\omega}^{\mathrm{PDL}}$ by a cut- and omega-rule free derivation. This in turn leads to a Herbrand-style conclusion that e.g. a given formula $\langle P^{*}\rangle A \vee F$ for star-free A and F is valid in **PDL** iff there exists a $k \geq 0$ and a cut- and omega-rule free derivation of a star-free sequent $A, \langle P\rangle^{1}A, \cdots, \langle P\rangle^{k}A, F$ where $\langle P\rangle^{i}A$ is an abbreviation for $\underbrace{\langle P\rangle \cdots \langle P\rangle}_{i\ times}A$. This holds particularly true for $\langle p^{*}\rangle A \vee F$ (p atomic program) being the negation of the EXPTIME-complete formula $\mathrm{ACCEPTS}_{M,x}$ expressing that any satisfying Kripke frame encodes an accepting computation of a given polynomial-space alternating Turing machine M on a given input x over M's alphabet. Thus the minimal valid star-free expansion $A, \langle p\rangle^{1}A, \cdots, \langle p\rangle^{k}A, F$ in question is derivable in the corresponding cut- and star-free (finite) fraction of $\mathrm{SEQ}_{\omega}^{\mathrm{PDL}}$ that in turn allows PSPACE proof search algorithm.