Discrete-Valued Sparse Signal

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Abstract

In many applications we are dealing with the problem of reconstructing discrete-valued sparse signals from a undetermined system of linear equations. One popular example is the detection of active users in a network, where usually most users are nonactive. Here, we basically need to recover a binary and sparse signal from few measurements, i.e. we want to recover $x \in \mathbb{R}^N$, from

$$Ax = y \quad \text{and} \quad \|x\|_0 \leq k \quad \text{and} \quad x \in \{0,1\}^N,$$

where $A \in \mathbb{R}^{m \times N}$, $y \in \mathbb{R}^m$, and $k \ll N$.

Whereas classical compressed sensing algorithms do not incorporate the additional knowledge of the discrete nature of the signal, classical lattice decoding approaches, like the sphere decoder, do not include the sparsity constraint.

In this talk, we will show first approaches to incorporate the additional structure information into orthogonal matching pursuit and basis pursuit. The basis pursuit approach will specifically deal with binary sparse signals, i.e sparse signals with entries in $\{0, 1\}$. We will show that phase transition appears earlier than by using the classical basis pursuit approach and that, independently of the sparsity of the signal, at most $N/2$ measurements are necessary to recover a binary signal uniquely, where N is the dimension of the ambient space. We will further discuss robustness and generalizations to signals with entries of a larger, than a binary, discrete set.