

# On nonstationary preconditioned iterative regularization methods for image deblurring

Alessandro Buccini\*and Marco Donatelli\*

## Abstract

In many applications, such as astronomy and medicine, arises the problem of image deblurring, this inverse problem is ill-conditioned and the inevitable presence of noise make a very difficult task obtaining a good reconstruction of the true image.

In the recent paper [3] the authors developed an algorithm for image deblurring which can be seen as an *approximated* version of the non-stationary iterated Tikhonov method. We call it approximated because we can obtain this method by substituting the true blurring operator, inside the Tikhonov iteration, with another one which is spectral equivalent but that has a structure that let us have very fast computations. Moreover, unlike the classical iterated Tikhonov, this method does not need the estimation of any parameter, which led to a more robust and stable algorithm.

In this talk we want to discuss some extensions of this method, in particular we add the possibility to project the reconstruction at each iteration onto a closed and convex set and to add a regularization operator in order to better preserve the edges (see [1]).

Recent results ([4]) have shown that iterative methods of multigrid type are very precise and efficient for regularizing purposes, thus a further extension is to include the proposed method in a multilevel strategy like the one in [2]. The algorithm in [2] was developed for Toeplitz matrices, i.e. for the zero boundary conditions, and combines multigrid methods with framelet denoise; the multigrid regularize and solve the problem and the denoise is used for keeping under control the effect of noise going on with the iterations.

Our proposal uses a multi-resolution representation of the point spread function to allow the method to be independent of the structure of the blurring matrix, in order to have the possibility to choose any boundary conditions and not only the zeros. Another important difference from [2] is the smoothing strategy: we propose to combine the previous nonstationary preconditioning and the projection into a closed and convex set.

Finally we will compare the results with the different approaches.

## References

- [1] A. BUCCINI *Regularizing preconditioners by non-stationary iterated Tikhonov with general penalty term*, Submitted, 2015.
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- [3] M. DONATELLI AND M. HANKE *Fast nonstationary preconditioned iterative methods for ill-posed problems, with application to image deblurring*, Inverse Problems 29, 2013.
- [4] M. DONATELLI AND S. SERRA-CAPIZZANO *On the regularizing power of multigrid-type algorithms*, SIAM J. Sci. Comput. 27–6 pp. 2053–2076, 2006.

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\*Dipartimento di Scienza e Alta Tecnologia, Università degli Studi dell'Insubria