On the derived cat. of global dim^{n} 2 j/w D. Ploog.

$$D^b_{\mathcal{H}}(A) \simeq \mathrm{Mat}_{\mathcal{H}}\left(\mathrm{Ext}^2(\mathcal{H}^{i+1}, \mathcal{H}^i)\right)$$

gl. dim 1.

fixed cohomology $\mathcal{H}$.

This isom. preserves indecomposables,
direct sums & summands
but morphisms might change

Motivation (Happel) $\mathcal{A} = \mathrm{Coh}\, X$, A-mod

sm. proj $\quad$ A f.d. alg $/ k = \bar{k}$

gl.dim $= 1$. X curve, A hereditary $(\mathrm{Ext}^2_A(M,N) = 0)$.

$$\mathrm{Ob}\, D^b(\mathcal{A}) = \{X^\bullet \mid d = 0\} \qquad X^\bullet \longmapsto \mathcal{H}^i(X^\bullet)$$

generalise this to gl.dim 2.

X surface: E exc., S spherical, phantoms, ...

concrete exa: S sph. obj in $\mathcal{A}$.

$\langle S \rangle$ gen. cat. $\quad$ Keller/Yang/Jorgenson

$d \geq 2$ $\quad \mathrm{Ext}^d(S,S) \simeq k \simeq \mathrm{End}(S) \qquad \mathrm{Ext}^i(S,S) = 0 \;\; \forall i \neq 0, d$

d=2: $\exists X^\bullet \quad H^i(X^\bullet) = \begin{cases} S & i \in [a,b] \\ 0 & i \notin [a,b] \end{cases}$

$D^b_{\mathcal{H}}(\mathcal{A}) = \{X \mid H^i(X^\bullet) \in \mathcal{H}^i\}$ additive cat.

$\mathcal{H} = \{\mathcal{H}^i\}_{i \in \mathbb{Z}}$ $\quad \mathcal{H}^i = \mathrm{add}\{H^{i1}, H^{i2}, \dots, H^{it(i)}\}$

indecomp. obj in $\mathcal{A}$.

$= \{\oplus (H^{i,j})^{a_{i,j}}\}$

$X^\bullet \longrightarrow Y^\bullet \longrightarrow Z^\bullet$ maps of cxs.

exact by def^n if $0 \to H^i(X^\bullet) \rightleftarrows H^\bullet(Y) \rightleftarrows H^\bullet(Z) \longrightarrow 0$

split exact.

ex X rational surface $\quad D \subseteq X$ prime div.

$D^2 = a$.

$\mathcal{C}_D = \langle \mathcal{O}(-D), \mathcal{O} \rangle$

$\mathbb{P}^2 \quad \langle \mathcal{O}(-1), \mathcal{O} \rangle \simeq \bullet \mathrel{\substack{\longrightarrow \\ \longrightarrow \\ \longrightarrow}} \bullet$

Thm [H-Ploog]

$D^2 \leqslant -1$. $\quad \langle \mathcal{O}(-D), \mathcal{O} \rangle \cap \mathrm{Coh}(X) \simeq \text{mod-}A$

$\mathcal{A} = \mathrm{Coh}(X) \cap \langle \mathcal{O}(-D), \mathcal{O} \rangle \simeq \text{mod-}A \qquad a \leq -1$

$a = -1 \qquad \bullet \longrightarrow \bullet \qquad A = k(\bullet \longrightarrow \bullet)$

3 indec. objs. [Happel].

$a = -2 \qquad \overset{1}{\bullet} \rightleftarrows \overset{2}{\bullet}$

5 indec. objs

Thm [H. Ploog] $\mathcal{D}^b_{\mathcal{H}}(\mathcal{A}) \xrightarrow{F} \mathrm{Mat}_{\mathcal{H}}\left(\mathrm{Ext}^2(\mathcal{H}^{i+1}, \mathcal{H}^i)\right)$

$X^\bullet \longmapsto \{H^i(X^\bullet), \xi_i\}$

If gl.dim $\mathcal{A} \leq 2$ then F is an equiv. of additive cats.