

Report on the Hausdorff Trimester Program

Complex Stochastic Systems: Discrete vs. Continuous

September 2007 - February 2008

Organizers: Sergio Albeverio, Jennifer Chayes, Gerard Ben Arous, Michel Ledoux, Karl Theodor Sturm

Topics

In recent years there is an increasing interest in application of stochastic models with a mixture of discrete and continuous elements. Their mathematical description and analysis often requires a combination of methods from combinatorics, stochastic analysis, partial differential equations, and geometric analysis. For example, discretization of the continuous components of a stochastic model is fundamental for numerical simulations. Conversely, continuum limits can be helpful to gain a better understanding of discrete models, e.g. in statistical mechanics. Powerful techniques, e.g. logarithmic Sobolev inequalities and concentration of measure estimates have originally been developed in a continuous setup, partially motivated by geometric considerations. Nowadays they are of rapidly increasing importance also in discrete setups, in the analysis of asymptotic and non-asymptotic issues of stochastic processes like the rate of convergence to equilibrium, and for various algorithmic applications. The extension of concepts from geometric stochastic analysis to singular spaces, graphs and random structures is a major challenge. A better understanding of the fascinating connections between random matrices and number theory requires a combination of probability, geometric analysis and algebra.

Goals

(i) There had been clear needs to increase significantly the interaction between stochastic analysis and non-linear partial differential equations on the one side, and discrete probability and theoretical computer science on the

other side. One aim of the trimester program was to bring together scientists from these different communities and to establish new collaborations and networks, in particular with existing research groups in Bonn.

(ii) A major goal of independent interest was to promote new developments within probability to a broad scientific community, in particular, in those areas which (at that time) had not been main stream of activities within Germany. Just to mention some hot topics of research: universality and random matrices, allocation and optimal transport, continuum mechanics and particle systems, mixing in stochastic algorithms.

The HIM has the chance to push developments and to stimulate future activities in new directions of research in Bonn as well as in Germany.

(iii) The trimester program was intended to create an inspiring atmosphere for researchers in the area and to stimulate discussions, exchange of ideas and new collaborations. The trimester program should provide an environment which allows to concentrate on research projects, without any distraction e.g. by administration or teaching.

(iv) Finally, emphasis is given to training of PostDocs and PhD students in this important field. This is reflected in a high number of introductory courses by leading top-level scientists.

Organization

The preparation of the trimester program was quite some challenge since the decision about formation and funding of HIM was made less than 12 months before the program was supposed to start. Moreover, half of the colleagues from the probability group at the IAM were about to retire within these period. In this situation, the network of outstanding mathematicians in Bonn with broad ranges of interest, locally well interacting, and perfectly linked with top scientists all over the world turned out to be one of the greatest assets of HCM.

Albeverio and Sturm arranged contact with leading experts in stochastic analysis, functional analysis, and mathematical physics (e.g. Malliavin, Kontsevich, Kotani, Pastur, Spohn), Eberle and Karpinski contact with top scientists in discrete probabilistic modeling, algorithms and random graphs (e.g. Borgs, Guionett, Jerrum, Peres), Otto and Sturm with prominent experts in PDEs and geometry (e.g. Villani, Funaki, Elworthy, Driver, Topping), and Albeverio and Sandmann with leading mathematicians in stochas-

tic modeling in finance (e.g. Björk, Kraft, Korn, Riedel, Mieltersen). In particular, Ben Arous, Chayes, and Ledoux could be convinced to join the local group as organizers of the whole trimester program. Moreover, we succeeded to attract numerous leading experts in the field as co-organizers of one of the five workshops and as lecturers of one of the nine introductory courses:

Mark Jerrum, London "Estimating the mixing time of Markov chains",
Sept. 10 - Sept. 21, 2007

Eric Carlen, New Brunswick "From particles to continuous mechanics",
Sept. 24 - Sept. 28, 2007

Maxim Kontsevich, Bures-sur-Yvette "Analytic aspects of Quantum
Field Theory", Oct. 15 - Oct. 24, 2007

Christian Borgs, Washington "Statistical Physics and Graph Theory:
From the Ising Model to Chromatic Polynomials and Markov Chains.",
Nov. 2 - Nov. 9, 2007

Anton Bovier, Berlin "Long term dynamics of disordered systems: from
metastability to ageing", Nov. 20 - Nov. 30, 2007

Alice Guionnet, Lyon "Lecture on Random matrices", Dec. 3 - Dec. 7,
2007

Martin Barlow, Toronto "Random walks and percolation", Dec. 12 -
Dec. 19, 2007

Yuri Kondratiev, Bielefeld "Interacting particle systems in continuum",
Jan. 21 - Feb. 1, 2008

Tomas Björk, Stockholm "Topics in interest rate theory", Feb. 11 - Feb.
15, 2008

The topics of these courses had been chosen to illustrate the various aspects and applications of the main theme of the whole program "Complex stochastic systems: discrete vs. continuous". Particular focus was on questions and problems arising in statistical physics. But also applications to quantum field theory, random graphs, algorithms, and economics had been considered. These topics had been taken up – in a different frame and under different aspects – in the workshops.

- W1 Stochastic processes and algorithms** (org. by J. Chayes, A. Eberle, M. Karpinski) September 3-7, 2007
- W2 Stochastic calculus on manifolds, graphs, and random structures** (org. by R. Bass, B. Driver, D. Elworthy, K.T. Sturm) October 8-12, 2007
- W3 Particle systems, nonlinear diffusions, and equilibration** (org. by G. Ben Arous, E. Carlen, F. Otto, K.T. Sturm, C. Villani) November 12-16, 2007
- W4 Random matrices** (org. by S. Alberverio, G. Ben Arous, K. Johansson, J. Keating, M. Ledoux)
Random matrices and number theory, January 11+12, 2008
Random matrices: probabilistic aspects and applications, January 14-18, 2008
- W5 Finance, Stochastics, Insurance** (org. by H. Kraft, K. Miltersen, J.A. Nielsen, K. Sandmann) February 25-29, 2008

Long-lasting results

(i) One of the impressive consequences of these trimester program is that it helped to establish permanently a new direction of research in Bonn, namely, random matrix theory. Previously, nobody in Bonn – and only very few people all over Germany – had been working in this area which in recent years became one of the most exciting and turbulent field of current research worldwide. One lecture series and one (extra large) workshop contributed to establish this as one of the new topics in applied mathematics in Bonn.

(ii) Moreover, the trimester program also strongly influenced the hiring of new colleagues in the probability group. It helped to present Bonn as one of the most fascinating places for mathematicians. The two new colleagues Bovier and Ferrari – who in the meantime succeeded Alberverio and Schäl – at that time had been participants of the trimester program.

(iii) The topic of the trimester program continues to be one of the challenging topics for future research. Competition and exchange between discrete and continuous models will also be a core topic of a proposal for a new SFB of the applied mathematicians at Bonn university.